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Numerical investigation of slow light in the coupled resonator optical waveguides using the local Fourier modal method

Junghyun Park[1], Hwi Kim[2], and Byoungho Lee[*, 1]

[1] National Creative Research Center for Active Plasmonics Application Systems, Inter-University Semiconductor Research Center and School of Electrical Engineering, Seoul National University, Gwanak-Gu, Gwanakro 599, Seoul 151-744, Korea
[2] Department of Electronics and Information Engineering, College of Science and Technology, Sejong Campus, Korea University, Jochiwon-eup, Yeongi-gun, Chungnam 339-700, Korea

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[*] Corresponding author: e-mail byoungho@snu.ac.kr, Phone: +82-2-880-7248, Fax: +82-2-873-9953

We present the Bloch modal analysis of photonic crystal coupled resonator optical waveguide (CROW) based on the local Fourier modal method (LFMM). The objective of this analysis is to extract the dispersion relation and analyze optical Bloch eigenmodes of the structure. The evolution of each Bloch eigenmode with varying frequency can be visualized with the proposed LFMM. The recently proposed CROW structure of NTT Basic Research Laboratory group with missing center holes and perturbed lateral shift of holes near the center is taken as the exemplary CROW structure.

1 Introduction Photonic crystal is an optical structure with photonic band structure formed by periodic refractive index distribution [1]. The solution to the Maxwell's equation in such a periodic structure is known to be composed of linear superposition of Bloch eigenmodes with a periodic wave envelope function and the Bloch phase term. Depending on the geometry of the photonic crystal, the Bloch wave number can be purely imaginary, indicating that light propagating through the photonic crystal feels it as opaque. This is referred to as the photonic bandgap. There has been considerable research on fundamental properties and practical applications based on the photonic crystal [2, 3].

By introducing various types of defect in the photonic crystal, a range of optical devices can be developed [2, 3]. It is well established that a row of missing holes or rods in photonic crystal shows the function of unprecedented effective wave-guiding. Since guiding mechanism of this waveguide results from the photonic bandgap, rather than the total internal reflection, it possess various interesting features such as low bending loss and small modal size. Optical resonators are also of interest due to its potential applications such as optical buffer and memory [4–6]. Photonic crystals with local defect result in the photonic crystal resonator [5, 6]. A dot or a cube of missing holes or rods can be used to trap light inside them. The small modal size and high quality (Q) factor of photonic crystal resonators have attracted lots of interest. Another method to form a photonic crystal resonator is to invoke small perturbation in lattice structures of the core of the photonic crystal waveguide [7]. Due to the broken symmetry of the lattice structure, there arises reflection of light propagating through the photonic crystal waveguide. Multiple reflections originating from periodic perturbations along the core give rise to another type of photonic crystal resonator. In addition, if the period of the cascaded structure photonic crystal resonators is large enough to partially isolate the overlap of wave function of the resonance mode and short enough to allow for coupling between neighbouring resonators, light can propagate through such a cascaded resonators, just like hopping. This is referred to as the coupled resonator optical waveguide (CROW) [7]. Considerable research has been devoted to the physical characteristics and the implement method of various kinds of CROWs [6, 7].

In order to analyze and design aforementioned photonic crystal-based optical devices, it is necessary to make use of an efficient computation algorithm. There are a number of numerical methods used in electromagnetics, such as the finite-difference time domain (FDTD) [8] or the finite element method (FEM) [9]. Since those numerical methods solve the Maxwell's equations in the real space domain, the memory space required to compute properties of photonic crystal-based optical devices is proportional to the size of them. However, the local Fourier modal method (LFMM) represents both the electromagnetic fields and the geometry in the spatial frequency domain [10, 11]. Consequently, the required memory space for several periods of the photonic crystal-based optical devices is much less compared to that in the FDTD or FEM. In particular, the overall response of CROWs can be calculated with the LFMM based S-matrix method and a simple algebraic relationship called the Redheffer star product. In this paper, we examine the slow light behaviour in the two dimensional CROW configuration by using the LFMM. The dispersion relation and corresponding field distributions will also be provided.

2 Theory Let us consider a multidimensional periodic structure with periods along the x, y, and z direction as Λ_x, Λ_y, and Λ_z, respectively. We choose a single computation supercell in a box with those periods. Optical responses observed from the outside of this supercell can be represented via the layer S-matrix, shown in Fig. 1. The elements of the matrix are given as $\vec{\mathbf{T}}$, $\overleftarrow{\mathbf{R}}$, $\vec{\mathbf{R}}$, and $\overleftarrow{\mathbf{T}}$, which correspond to the transmission and reflection operators under the left-to-right and right-to-left directional characterization, respectively [10, 11].

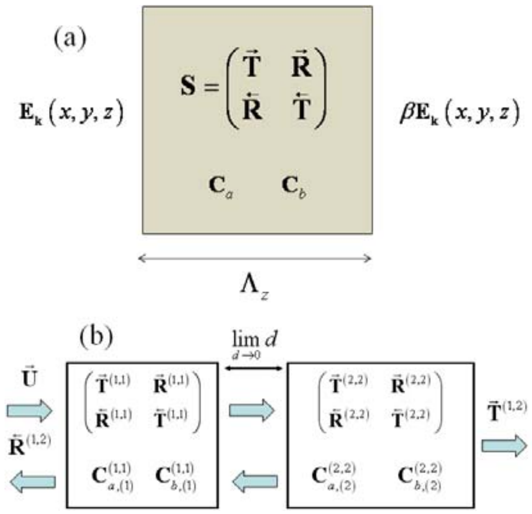

Figure 1 (Color online) (a) Layer S-matrix and Bloch eigenmode with the eigenvalue $\beta = \exp\left(jk_{z,0}\Lambda_z\right)$. (b) Interconnection of two single blocks via the Redheffer star product.

Once the layer S-matrix in a single computation cell is obtained, the layer S-matrix of composite multilayer cells can be easily obtained by using the Redheffer star product as follows [10].

$$\overleftarrow{\mathbf{R}}^{(1,2)} = \overleftarrow{\mathbf{R}}^{(1,1)} + \overleftarrow{\mathbf{T}}^{(1,1)}\left(\mathbf{I} - \overleftarrow{\mathbf{R}}^{(2,2)}\vec{\mathbf{R}}^{(1,1)}\right)^{-1}\overleftarrow{\mathbf{R}}^{(2,2)}\vec{\mathbf{T}}^{(1,1)}, \tag{1}$$

$$\vec{\mathbf{T}}^{(1,2)} = \vec{\mathbf{T}}^{(2,2)}\left(\mathbf{I} - \vec{\mathbf{R}}^{(1,1)}\overleftarrow{\mathbf{R}}^{(2,2)}\right)^{-1}\vec{\mathbf{T}}^{(1,1)}, \tag{2}$$

$$\vec{\mathbf{R}}^{(1,2)} = \vec{\mathbf{R}}^{(2,2)} + \vec{\mathbf{T}}^{(2,2)}\left(\mathbf{I} - \vec{\mathbf{R}}^{(1,1)}\overleftarrow{\mathbf{R}}^{(2,2)}\right)^{-1}\vec{\mathbf{R}}^{(1,1)}\overleftarrow{\mathbf{T}}^{(2,2)}, \tag{3}$$

$$\overleftarrow{\mathbf{T}}^{(1,2)} = \overleftarrow{\mathbf{T}}^{(1,1)}\left(\mathbf{I} - \overleftarrow{\mathbf{R}}^{(2,2)}\vec{\mathbf{R}}^{(1,1)}\right)^{-1}\overleftarrow{\mathbf{T}}^{(2,2)}, \tag{4}$$

where superscript (1,1), (2,2), and (1,2) denote the left single-block, the right single-block, and the multi-block, respectively. The electric $\tilde{\mathbf{E}}_{\mathbf{k}}$ and magnetic $\tilde{\mathbf{H}}_{\mathbf{k}}$ fields are composed of the Bloch phase term and periodic envelop functions $\mathbf{E}_{\mathbf{k}}$ and $\mathbf{H}_{\mathbf{k}}$ as follows [10].

$$\tilde{\mathbf{E}}_{\mathbf{k}}\left(x,y,z\right) = \exp\left[j\left(k_{x,0}x + k_{y,0}y + k_{z,0}z\right)\right]\mathbf{E}_{\mathbf{k}}\left(x,y,z\right), \tag{5}$$

$$\tilde{\mathbf{H}}_{\mathbf{k}}\left(x,y,z\right) = \exp\left[j\left(k_{x,0}x + k_{y,0}y + k_{z,0}z\right)\right]\mathbf{H}_{\mathbf{k}}\left(x,y,z\right), \tag{6}$$

where $k_{x,0}$, $k_{y,0}$, and $k_{z,0}$ represent the x, y, and z-components of the Bloch wave vector, respectively. For given eigenvalue $\beta = \exp\left(jk_{z,0}\Lambda_z\right)$ the electromagnetic eigenmode can be expressed as $\left(\vec{w}\ \overleftarrow{w}\right)^T$, where $\vec{w}$ and $\overleftarrow{w}$ are the Fourier spectra of the right-direction and the left-direction propagating portions of the Bloch eigenmode. According to the Bloch theorem, the eigenmode experiences only the Bloch phase shift of β under translation of $z \to z' = z + \Lambda_z$. This relation can be shown as the Bloch mode condition as follows [10].

$$\begin{pmatrix} \vec{\mathbf{T}}^{(1,2)} & \mathbf{0} \\ \vec{\mathbf{R}}^{(1,2)} & -\mathbf{I} \end{pmatrix}\begin{pmatrix} \vec{w} \\ \overleftarrow{w} \end{pmatrix} = \beta\begin{pmatrix} \mathbf{I} & -\overleftarrow{\mathbf{R}}^{(1,2)} \\ \mathbf{0} & -\overleftarrow{\mathbf{T}}^{(1,2)} \end{pmatrix}\begin{pmatrix} \vec{w} \\ \overleftarrow{w} \end{pmatrix}. \tag{7}$$

By solving the eigenvalue problem given in Eq. (7), we can obtain the Bloch wave number and corresponding Bloch eigenmodes for any periodic structures.

3 Coupled resonator optical waveguide Figure 2 shows a schematic diagram of the exemplary photonic crystal CROW composed of a triangular photonic crystal structure with a row of missing air holes along the center line [6]. The lattice constant a and the radius r of air holes are 420 nm and 180 nm, respectively. By introducing tapered lateral shifts to the position of air holes near the waveguide center, the optical resonator is formed. The distance of lateral shifts are 30 nm, 20 nm, and 10 nm for the air holes of type A, B, and C, respectively. The refractive indexes of dielectric and air are 3.46 and 1.00, respectively.

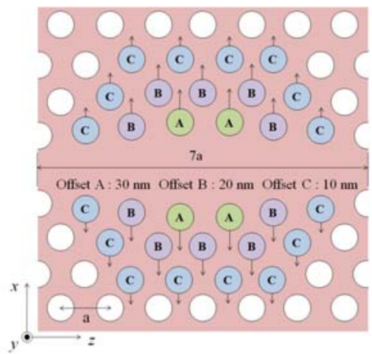

Figure 2 (Color online) Schematic diagram of photonic crystal coupled resonator optical waveguide

This kind of optical resonators are cascaded along the *z*-direction with the period of 7*a*. It is assumed that there is no *y*-dependence. The transverse magnetic (TM) polarized light is incident from the left side.

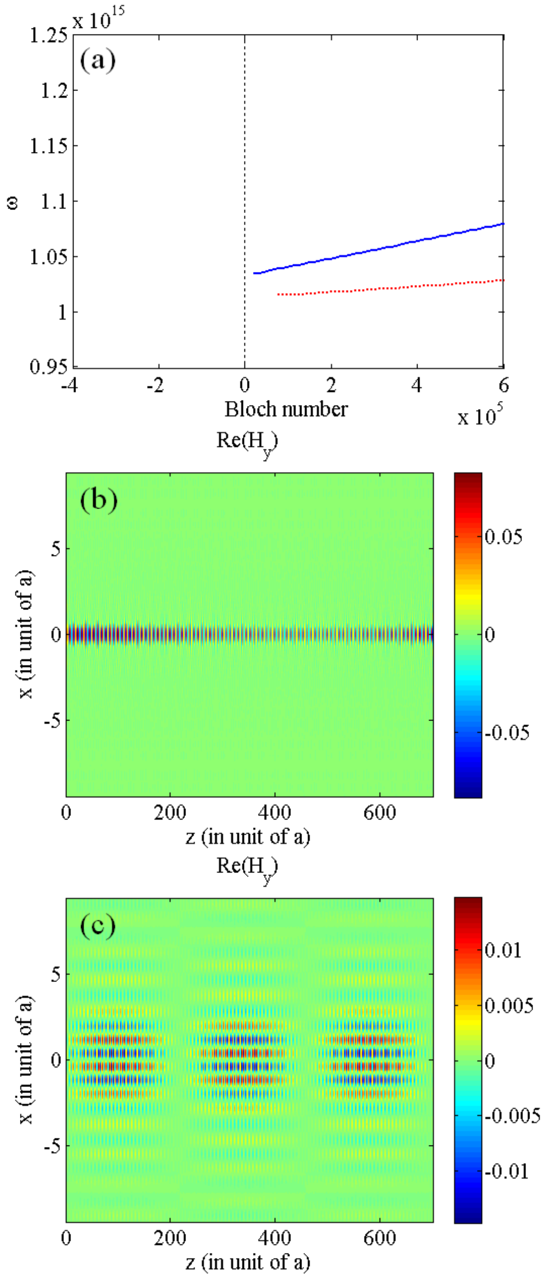

Figure 3 (Color online) (a) Dispersion relation of the photonic crystal CROW shown in Fig. 2. (b) and (c) H_y field distributions for the band of the solid blue and dotted red lines in (a), respectively.

The Bloch wave number β is obtained by solving Eq. (7), which is dependent on the operating angular frequency ω.The relationship between β and ω, i.e., the dispersion relation is plotted in Fig. 3(a). It is observed that there are two Bloch eigenmodes propagating along the CROW: one follows the solid blue (upper) line and the other the dotted red (bottom) line. The H_y field distribution for the upper and bottom modes are depicted in Figs. 3(b) and 3(c), respectively. The abscissa *z* and the ordinate *x* are plotted in unit of the photonic crystal period *a*. In Fig. 3(b), we observe a highly confined optical mode. However, this is not related to the Bloch mode of slow light. We note the bottom mode with low group velocity. Note that the field profile extends to deep cladding region. The group index obtained from the slope of the dispersion relation of this mode was calculated to be 11.4. This slow light phenomenon is ascribed to the fact that weak coupling between neighbouring optical resonators is formed along the photonic crystal waveguide.

4 Conclusion The photonic crystal CROW with extremely slow light is investigated by adopting the LFMM algorithm. The Bloch modal analysis based on the S-matrix method and the LFMM is presented. The eigenvalue problem formulated by the LFMM yields the intrinsic Bloch wave number. Since the LFMM deals with the Maxwell's equations in the spatial-frequency domain and exploits the periodicity of CROW structure efficiently, it provides a complete modal solution and does not suffer from the increase of the required memory.

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References

[1] J. D. Joannopoulos, P. R. Villeneuve, and S. Fan, Solid State Commun. **102**, 165 (1997).
[2] A. Mekis, J. C. Chen, I. Kurland, S. Fan, P. R. Villeneuve, and J. D. Joannopoulos, Phys. Rev. Lett. **77**, 3787 (1996).
[3] T. F. Krauss, J. Phys. D: Appl. Phys. **40**, 2666 (2007).
[4] K.-Y. Kim and S. Kim, J. Opt. Soc. Korea **13**, 484 (2009).
[5] Y. Akahane, T. Asano, B.-S. Song, and S. Noda, Nature **425**, 944 (2003).
[6] T. Tanabe, M. Notomi, E. Kuramochi, A. Shinya, and H. Taniyama, Nature Photon. **1**, 49 (2007).
[7] A. Yariv, Y. Xu, R. K. Lee, and A. Scherer, Opt. Lett. **24**, 711 (1999).
[8] A. Taflove and S. C. Hagness, Computational Electrodynamics: The Finite-Difference Time-Domain Method, 2nd ed. (Artech House, Norwood, 2000).
[9] J. N. Reddy, An Introduction to the Finite Element Method, 2nd ed. (McGraw Hill, New York, 1993).
[10] H. Kim and B. Lee, J. Opt. Soc. Am. B **25**, 518 (2008).
[11] H. Kim and B. Lee, J. Opt. Soc. Am. A **25**, 40 (2008).

