

Coupling of spin and angular momentum of light in plasmonic vortex

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Abstract: We present that two distinct optical properties of light, the spin angular momentum (SAM) and the orbital angular momentum (OAM), can be coupled in the plasmonic vortex. If a plasmonic vortex lens (PVL) is illuminated by the helical vector beam (HVB) with the SAM and OAM, then those distinct angular momenta contribute to the generation of the plasmonic vortex together. The analytical model reveals that the total topological charge of the generated plasmonic vortex is given by a linear summation of those of the SAM and OAM, as well as the geometric charge of the PVL. The generation of the plasmonic vortex and the manipulation of the fractional topological charge are also presented.

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1. Introduction

Surface plasmons (SPs) are surface electromagnetic waves coupled with collective oscillation of electrons at metal-dielectric interfaces [1–3]. There has been considerable research devoted to the efficient excitation of SPs [4, 5], focusing SP [6, 7], plasmonic waveguides [8, 9], and generation and manipulation of plasmonic hot spots [6, 7, 10–13]. Furthermore, formation of SP interference pattern on metal surface using metal grating and laser beam is also a hot issue in this field [14–17]. The diffractive SP pattern is influenced by laser beam profile and condition of metal-dielectric interface, such as shape of grating and refractive indexes of metal and dielectric.

Considerable research has been devoted to fundamentals and applications for SP interference pattern based on the angular momenta of light [18–23]. Spin angular momentum (SAM) resides in the circularly polarized beam, whereas orbital angular momentum (OAM) comes from the spiral phase profile of beam around the axis of propagation. In our previous study, effect of laser beam with SAM and its application for plasmonic vortex lens (PVL) structure were presented [17]. Although there are a few researches about the coupling of the PVL and each of the angular momentum [24], discussion on the overall effect of the SAM, the OAM and the geometrical charge of the PVL to the plasmonic vortex has not yet been established.

In this paper, the overall effect of the two angular momenta and the PVL structure on SP interference patterns is examined. Explanations on the helical vector beam (HVB) with two angular momenta are given. We propose a method to generate the plasmonic vortex with arbitrary topological charge, synthesized with momenta of HVB and geometrical effect of PVL. It is shown that effects of the SAM and the OAM of light can be coupled in the plasmonic vortex.

2. Helical vector beam

The electromagnetic field component of an HVB has an azimuthal phase term of $\exp(il\phi)$. Here, l is the helix winding number of 2π cycles in phase around the circumference. Light with the helix winding number l is regarded as a flux of photons with the OAM $l\hbar$. This azimuthal phase distribution can be obtained by the spiral phase plate or the spatial light modulator (SLM). Note that the positive and negative signs of l correspond to the direction of counter-clockwise and clockwise rotating electromagnetic fields, respectively. There are various kinds of optical beam with helicity such as the Laguerre-Gaussian beams and the Bessel beam [25].

The transverse field of light can be expressed as a superposition of two orthogonal electric fields. Depending on the amplitude and phase relationship between them, various polarization states can be obtained such as linearly, circularly, and elliptically polarized beams. Those states are achieved by using the half- or quarter-wave plates. Especially, the circularly polarized beam is considered to have the SAM of $\sigma = s\hbar$, where $s = +1$ for the right-handed circular polarization (RCP) and $s = -1$ for the left-handed circular polarization (LCP), respectively [15]. There are other nonconventional polarization states of light that have the electric field direction with spatial dependence, such as radially and azimuthally polarized beams [15]. Corrugated surface gratings or specific polarizer can be used to generate those polarization states of light [26]. The magnetic fields of radially and azimuthally polarized beams are determined to be orthogonal to the electric fields, so that the resultant magnetic fields are azimuthally and radially polarized, respectively.

The two properties of light mentioned above can be combined together, leading to the HVB. In particular, the phase and polarization state can be independently controlled to have arbitrary combinations by cascading wave plates and spiral phase plates. Figure 1 represents electric field distributions at a fixed time for various combinations of polarization states and helix winding numbers. The top panel corresponds to the LCP, the middle the radial polarization, and the bottom the RCP. From the left panel, the helix winding number l varies from -1 to 2 with increment of 1 . The abscissa and the ordinate denote the x and y coordinates with arbitrary unit, respectively. In addition to the polarization factor, the azimuthal phase term of $\exp(il\phi)$ is multiplied as well as the radial term of $J_j(\rho)$, where J_j and ρ are the j th-order Bessel function with the first kind and the radial coordinate ($\rho = (x^2 + y^2)^{1/2}$). Even though the polarization is different for each panel, they look quite similar in some cases. For example, the LCP with $l = 1$ (Fig. 1(c)), the radial polarization with $l = 0$ (Fig. 1(f)), and the RCP with $l = -1$ (Fig. 1(i)) look similar at a fixed time. However, the field distributions with time development show definitely unique patterns. We thus present a movie clip for Fig. 1 that clearly shows the electric field distribution with varying time. In the movie clip, the handedness symmetry is observed for two combinations: the LCP with $l = 1$ (Fig. 1(c)) and the RCP with $l = -1$ (Fig. 1(i)), and the LCP with $l = -1$ (Fig. 1(a)) and the RCP with $l = 1$ (Fig. 1(k)). In addition, it will be shown below that there are such combinations that result in the same topological charges due to coupling of the spin and angular momenta in the PVL. From these observations, an important remark can be made that the polarization states and helix winding number of phase can be separately manipulated so that we obtain any arbitrary combination of SAM and OAM.

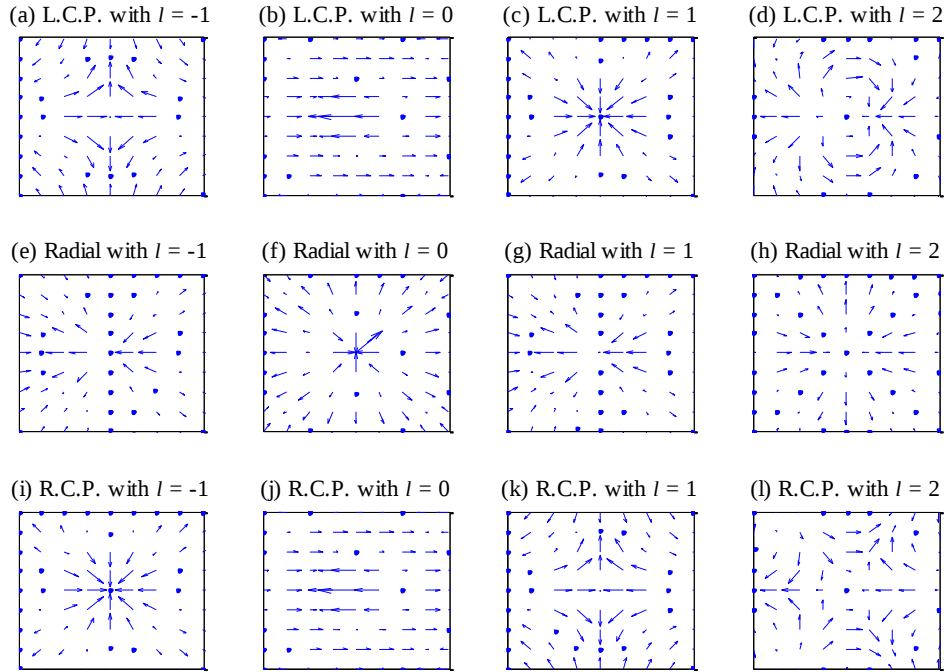


Fig. 1. Electric field distributions at a fixed time for various combinations of polarization states and helix winding numbers (Top panel: the LCP, middle panel: the radial polarization, bottom panel: the RCP). The helix winding number l varies from -1 to 2 with increment of 1 (Media 1).

3. SP excitation at metal slit

Now let us discuss about the excitation mechanism of SPs based on the subwavelength metal slit. When the slit is illuminated by light from one side of metal, light passing through the slit excites SP wave on the other side. We note that the normal incidence of light induces the equal phase distribution of electron oscillations across the slit and thus the propagation direction of SPs generated by slit is always perpendicular to the slit. The electric field component normal to the slit contributes to the longitudinal oscillation of electrons. Consequently, the magnitude of excited SP becomes the maximum when the polarization of incident light has the electric field perpendicular to the slit.

We illustrate in Fig. 2 three cases of SP excitation at a slit with the back-side illumination. Figure 2(a) shows the case that the polarization of illuminating light is perpendicular to the slit. In this case, the direction of polarization is the same as that of direction of SPs propagation. Therefore, the electric field thoroughly contributes to SP excitation. Second case is shown in Fig. 2(b), in which the direction of the polarization does not coincide with direction of SPs propagation, having a nonzero angle. When the polarization has such nonzero angle, energy of electric field cannot thoroughly contribute to SP excitation. Rather the projection of electric field direction upon SP propagation direction can excite SPs. The third case is that the slit is illuminated by HVB, as shown in Fig. 2(c). In contrast to the previous two cases, the HVB exhibits time-varying and spatially-dependent polarization direction. As a result, the angle between directions of the polarization and SP propagation varies along the slit.

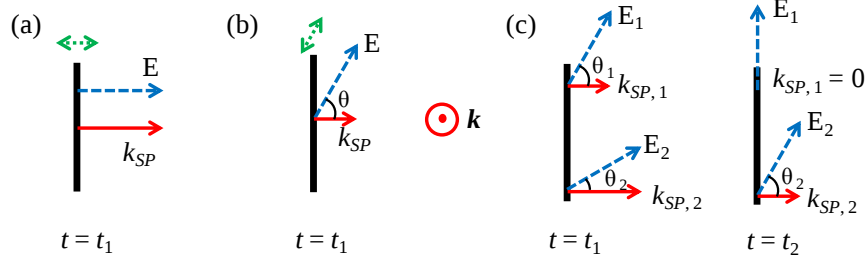


Fig. 2. Three cases of SP excitation at a slit (thick black line) with the backside illumination. The laser with the propagation wave vector k coming out of the paper illuminates bottom of slit and excites SPs on front-side of slit. (a) The polarization of illuminating light is perpendicular to the slit. (b) The direction of the polarization having a nonzero angle to the direction of SPs propagation. (c) The slit is illuminated by the HVB.

We advance our analogy to a more complicated case where the slit is not a single straight line anymore but has a complex pattern. In particular, the PVL under illumination of the HVB is considered. The PVL is composed of multiple spiral slits to form spiral phase retardation of SPs propagating from the slits to the center of the geometry. The phase retardation is controlled by the distance from the center of the PVL to the slits. The PVL forms plasmonic vortex patterns that have co-centric circles with centers at the center of the geometry. The nearest circle pattern from the center is called primary ring [20]. The geometry of the PVL with spiral slits can be described in a simple algebraic form. The distance r from the center of the PVL to the slit with the azimuthal angle ϕ is given by $r = a + (\lambda_{sp} m \phi) / (2\pi)$. Here, a is the inner radius of the PVL, m is the geometrical charge of the PVL, and λ_{sp} is the effective wavelength of SPs. According to this equation, the distance from slit to the center is proportional to the value of azimuthal angle ϕ . Since the energy of SPs decays as SPs propagate, SP propagating from slits at large azimuthal angle experiences large energy decay and the uniformity of SP energy at the center deteriorates. In order to avoid this unnecessary degradation, the equation above can be modified to $r = a + \lambda_{sp} \text{mod}(m\phi, 2\pi) / (2\pi)$, where the notation $\text{mod}(x, y)$ represents the remainder of the division of x by y . The PVL slit pattern based on the modified equation has m partial slit segments and the distance between the center of geometry and slits is in a range from a to $(a + \lambda_{sp})$, providing a moderate uniformity of SP energy.

When the PVL slit pattern is illuminated by the HVB from the back side, the SPs excited at the metal slit propagate toward the center of the PVL and generate the SP vortex. The properties of the SP vortex are governed by the total topological charge. In order to understand how the total topological charge is determined, it is necessary to examine the phase relationship in the PVL. Figure 3 presents the phase diagram of SPs excited at the PVL slits for various cases. The phase of SPs is highly dependent on the relationship between the direction of the instantaneous electric field and the direction of infinitesimal metal slit for the PVL. Throughout this paper, we define the relative phase Ω of the excited SPs as zero or an integer multiple of 2π when the amplitude A_{sp} of the in-plane electric field $E_{\parallel}$ of incident light toward the center of the PVL reaches its maximum ($\Omega = \angle A_{sp}$). Here $A_{sp} = [\mathbf{E}_{\parallel} \cdot (-\hat{\mathbf{r}})] / |\mathbf{E}_{\parallel}|$ and $\mathbf{r} = x \cos \phi + y \sin \phi$. Fig. 3(a) shows the phase diagram for the radial polarization with $l = 0$. For the initial state with $\omega t = 0$, the in-plane electric field heads toward the center of the PVL. The resultant phase is $\Omega = 0$, which is denoted by the red arrows. As time evolves, the phase decreases and the phase becomes $\Omega = 3\pi/2$, (or $\Omega = -\pi/2$), which is represented by the yellow arrows. The next level ($\omega t = \pi$) exhibits the

in-plane electric field directed outward of the PVL and the resultant phase is $\Omega = \pi$ (the blue arrows). The final phase with $\omega t = 3\pi/2$ gives rise to $\Omega = \pi/2$. Note that all the phases Ω are the same along the circle in this case. In Figs. 3(b)-3(d), we show more complicated cases. If there is a phase retardation coming from the helical winding ($l = 1$, Fig. 3(b)), then the SPs excited at the slit also carry the phase retardation proportional to $\exp(i\phi)$. If the PVL is illuminated by the RCP without the helical phase retardation (Fig. 3(c)), the initial phase of SPs excited at the slit depends on the azimuthal phase. This is because the angle between the directions of the slit and the in-plane electric field differs along the azimuthal phase of the slit position. Figure 3(d) illustrates the PVL with $m = 1$ illuminated by the full-symmetric HVB with the radial polarization and no helical phase. Note that the initial phases at the slit are the same. In this case, however, the distance from the center of the PVL to the slit is linearly proportional to the azimuthal angle of the slit. Therefore, another type of phase retardation occurs: the propagation phase retardation. This relation is represented by the arrows between the PVL slit and the hypothetical circle (denoted by dashed line). Note that the number of arrows is dependent on the azimuthal angle. The resultant phase distribution along the dashed circle exhibits the same phase distribution as those of the case in Figs. 3(b) and 3(c). From discussion above based on the phase diagram, it can be inferred that the phase distribution of the SPs inside the PVL is affected by three factors: the helical phase of the HVB, the polarization state, and the geometrical charge of the PVL.

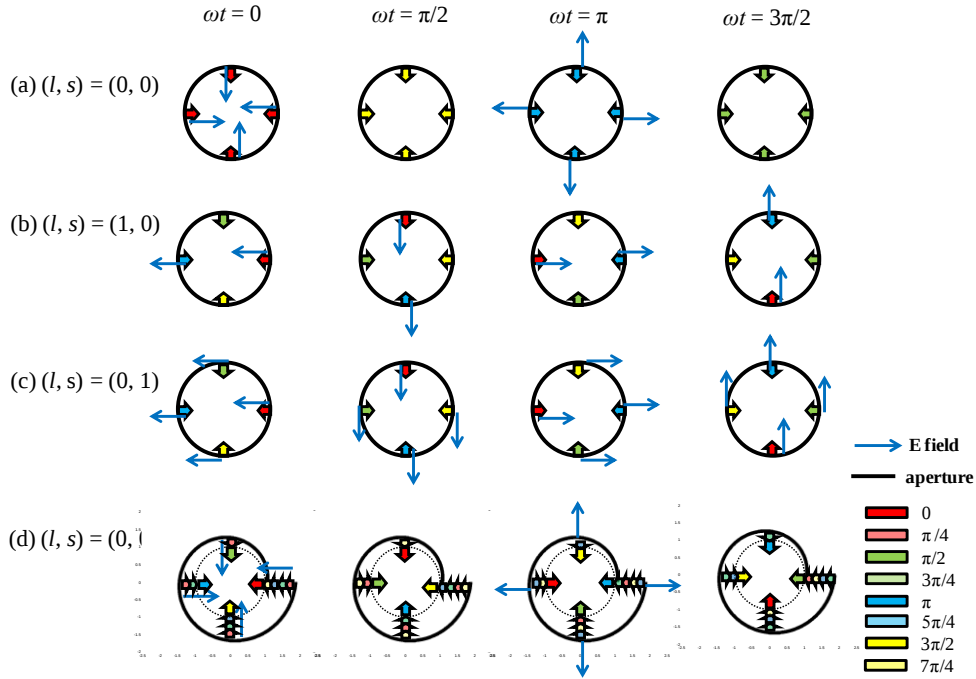


Fig. 3. Phase diagram of SPs generated at the PVL for four cases: (a) $(l, m, s) = (0, 0, 0)$, (b) $(l, m, s) = (1, 0, 0)$, (c) $(l, m, s) = (0, 0, 1)$, and (d) $(l, m, s) = (0, 1, 0)$. The most left panel corresponds to the phase of $\omega t = 0$. As time evolves, each phase decreases since wave functions have the time-dependence of $\exp(-i\omega t)$. The most right panel exhibits the phase of $\omega t = 3\pi/2$.

The aforementioned property can be expressed in a simple algebraic equation. When the HVB with azimuthal phase term $\exp(il\phi)$ illuminates the PVL, the phase on the slit varies according to the position on the PVL. The value of topological charge can be either increased

or decreased by proper helical phase. Here phase term of $\exp(il\phi)$ shares the same positive direction as that of geometrical charge of the PVL. The topological charge j by the PVL and the HVB is given by $j = l + m$. In addition, this can be extended to take the effect of polarization status into account, i.e., the total topological charge j is given by

$$j = l + m + s. \quad (1)$$

One may be inclined to ask the values of spin parameter s for the radial and azimuthal polarizations. In the PVL structure, the radially polarized HVB excites SPs as seen for the linear polarization perpendicular to slit at each point. s of the radial polarization is 0 as can be observed with simulation results in Section 4.

4. Numerical simulation

The aforementioned hypothesis is verified by using full-vectorial and three-dimensional rigorous coupled-wave analysis (RCWA) [27]. In the simulation it is assumed that the experimental setup is as shown in Fig. 4(a). The PVL is inscribed on the gold layer. Figure 4(b) shows the schematic of the PVL whose m is 1. The wavelength of laser is 660 nm and the corresponding wavelength of SPs is 629 nm. The slit width is 250 nm, thickness of gold layer is 300 nm, and inner radius, a , is $4 \mu\text{m}$. The laser beam is assumed to be modulated by the phase SLM. When the modulated laser beam passing through a quarter wave plate or a radial polarizer illuminates the PVL from the backside, SPs are excited at the slit and propagate toward the center of the geometry. The simulation is carried out with altering three variables: the variable for geometrical effect of the PVL m , the polarization effect s , and the helix winding number of azimuthal phase term l . The key factor governing the properties of the plasmonic vortex is the topological charge. We can extract the topological charge by measuring the primary ring size of the generated plasmonic vortex [17].

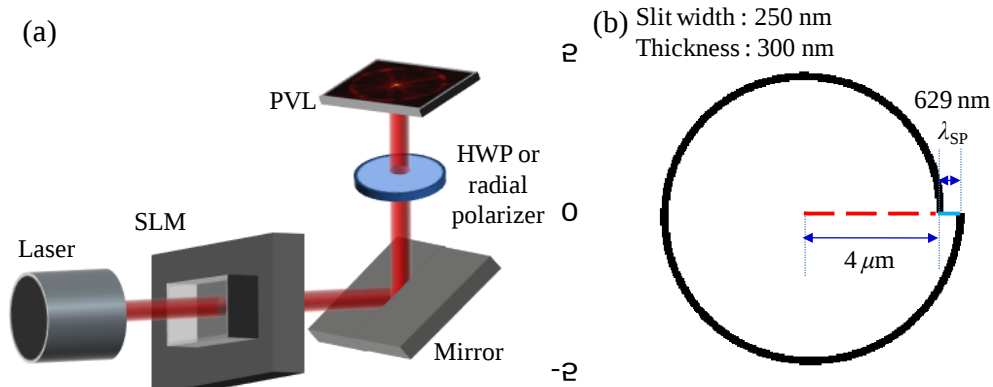


Fig. 4. (a) Hypothetical experimental setup. The PVL is inscribed on the gold layer. The wavelength of laser is 660 nm and the corresponding wavelength of SPs is 629 nm. (b) The schematic of spiral slit ($m = 1$) and design parameters of the PVL are shown.

Figure 5 shows the simulation result with various HVBs with the fixed geometrical charge ($m = 1$) which is shown in Fig. 4(b). In this figure, the top panel corresponds to the LCP ($s = -1$), the middle to the radial polarization ($s = 0$), and the bottom to the RCP ($s = +1$). From the left panel, helix winding number l varies from -1 to 2 with increment of one. For the sake of clear comparison, the arrangement of panels in Fig. 5 is the same as that in Fig. 1. As shown in Fig. 5, for the PVLs with the topological charge of 2, such as $(l, m, s) = (1, 1, 0)$, $(0, 1, 1)$, and $(2, 1, -1)$, the primary ring sizes of the plasmonic vortices are the same. Other PVL group with the same topological charge exhibited the same vortex radius as expected.

Figure 6 shows the geometric effect of PVL. While the polarization parameter s and the helix winding number l are fixed, the geometric effect m of the PVL is varied. It turns out that the change of the primary ring sizes of the plasmonic vortex also depends on the variation in geometrical charge of PVL as shown in Fig. 6(a) with $(l, m, s) = (0, 1, 0)$ and Fig. 6(b) with $(l, m, s) = (0, 3, 0)$. In this case the total vortex charges are different from each other, resulting in the different sizes of the plasmonic vortices. However, even if the sum of the polarization effect and the helix winding number is different, vortex radii can be the same when PVLs are appropriately used as shown in Fig. 6(c) with $(l, m, s) = (-1, 3, -1)$. We note that the primary ring sizes of the plasmonic vortices in Figs. 6(a) and 6(c) are the same. This is because the total vortex charges of those configurations are the same ($j = 1$).

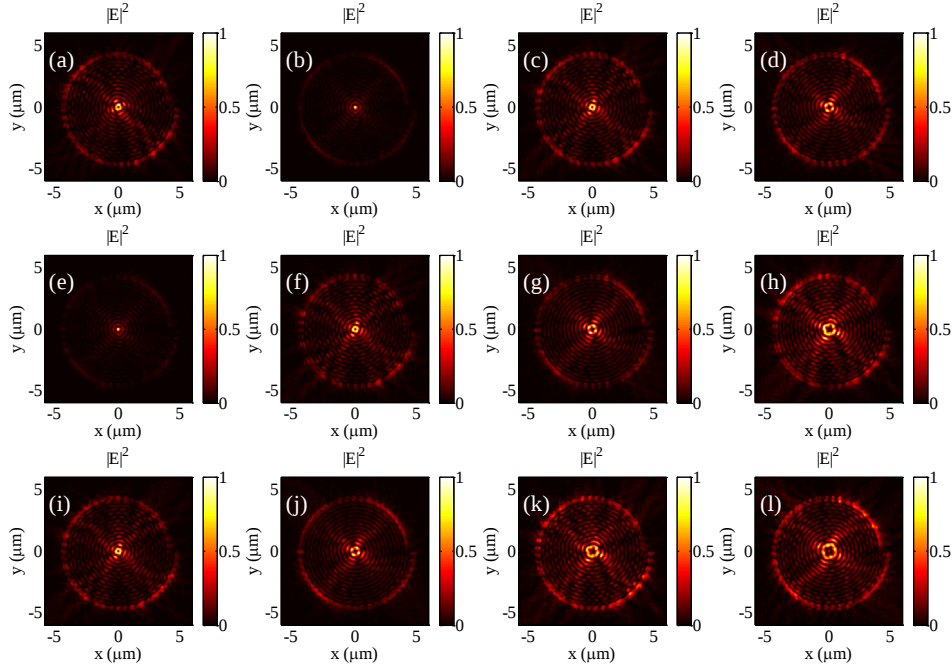


Fig. 5. Simulation result with various HVBs for a fixed geometrical charge ($m = 1$, which is shown in Fig. 4(b)). For the sake of clear comparison, the arrangement of panels is the same as that in Fig. 1.

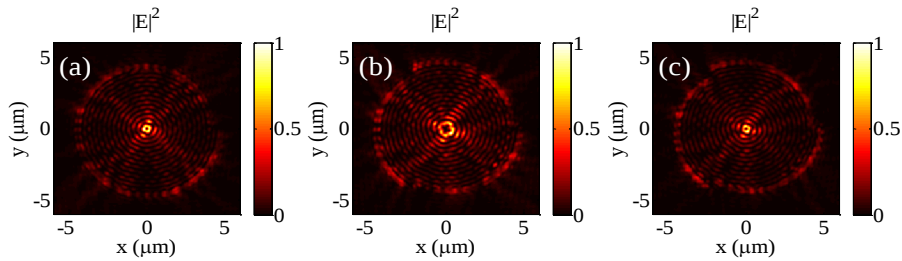


Fig. 6. Geometric effect of PVL. (a) $(l, m, s) = (0, 1, 0)$, (b) $(l, m, s) = (0, 3, 0)$, (c) $(l, m, s) = (-1, 3, -1)$.

The quantitative analysis of the result is as below. Point sources Q_{SRC} are aligned along the circle with the radius b and the angle ϕ . They are assumed to have the phase distribution function $g(\phi)$ which implies overall effects of the geometric, polarization, and azimuthal phases. The SPs propagate from a point source with the SP propagation constant k_{SP} . When

an observation point is represented by $P(r, \phi)$ inside the PVL, the wave function $f(r, \phi)$ at $P(r, \phi)$ is given by

$$f(r, \phi) = \int_0^{2\pi} g(\phi) \exp[jk_{sp} |P(r, \phi) - Q_{SRC}(b, \phi)|] d\phi. \quad (2)$$

By using Taylor's expansion and the integral form of the Bessel function of the first kind, we obtain approximation equation,

$$f(r, \phi) \approx 2\pi J_j(k_{sp}r) \exp(ik_{sp}b) \exp[ij(\phi - \pi/2)], \quad (3)$$

where $j = l + m + s$. The quantitative plasmonic vortex pattern is given with intensity of electrical field, $|f(r, \phi)|$. Therefore, the primary ring size is given with solution of the Bessel function of the first kind.

Figure 7 shows the radius of vortex as a function of the azimuthal phase term l . Each figure displays different geometric effect, such as $m = -1$ in Fig. 7(a) and $m = 0$ in Fig. 7(b). The blue dashed, green dotted, and red solid lines with diamond, cross, and circle markers denote results of the RCP, the radial polarization, and the LCP, respectively. The left vertical axis of ordinates displays radius of vortex and the right vertical axis, solution of Bessel function of the first kind. As expected, the result shows the radii of vortices are identical when the PVLs have same topological charge j , which is the sum of geometrical charge m , polarization effect s , and azimuthal phase term l . Moreover, this figure shows that the primary ring sizes of simulation result are almost identical to solution of the Bessel function of the first kind.

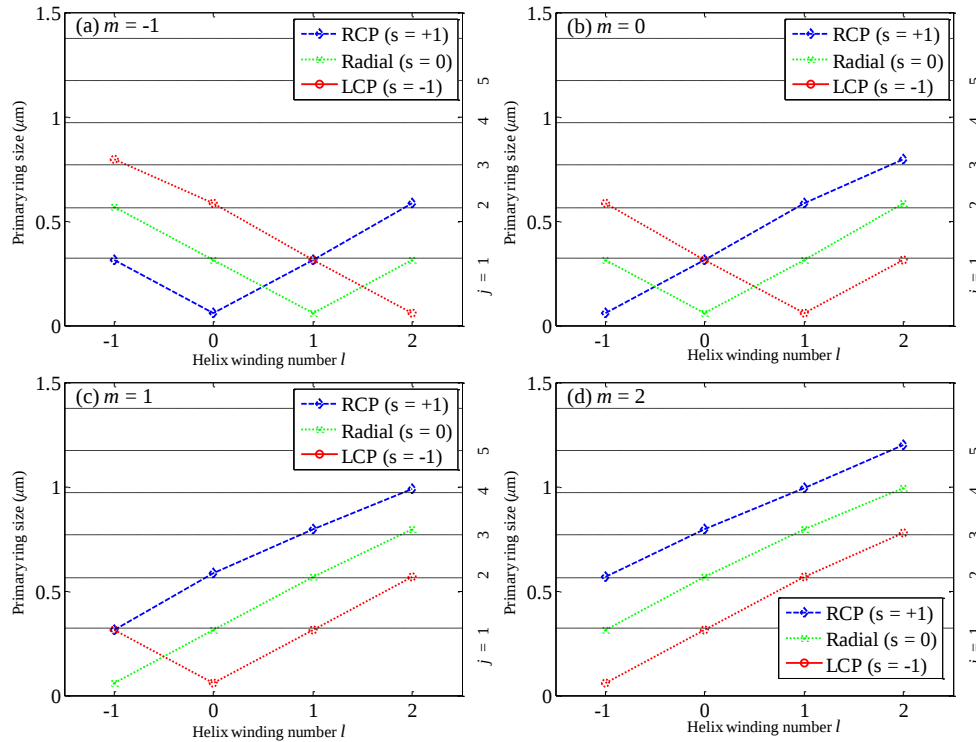


Fig. 7. Radius of vortex as a function of azimuthal phase term l . Each figure displays different geometric effect, (a) $m = -1$, (b) $m = 0$, (c) $m = 1$, (d) $m = 2$. The blue dashed, green dotted, and red solid lines with diamond, cross, and circle markers denote results of the RCP, the radial polarization, and the LCP, respectively. The black dashed lines denote solution of the Bessel function of the first kind.

When the parameters l , m and s are given, we can estimate the primary ring size. On the contrary, even though if we know the primary ring size of vortex, it is difficult to individually estimate the parameter l , m and s . The geometrical charge m can be easily obtained by using microscope. When l is 0, the method to find polarization of illuminated laser using PVL whose m is 1 is reported [28]. In case l is non-zero, however, it is difficult to distinguish l and s from $l + s$ because l and s equally contribute to deciding the primary ring size.

Here, one may be inclined to ask the maximum primary ring size that can be excited by the PVL. If the primary ring size exceeds the diameter of the PVL, then the SP vortex field inside the PVL vanishes. Thus it appears that the topological charge in practical applications should be limited so that the primary ring appears inside the PVL with a finite diameter. In order to obtain higher orbital angular momentum of vortex, we can increase both the diameter of a PVL and the topological charge j . However, the propagation loss of the SP wave becomes dominant for a PVL with a large diameter [6]. Therefore it seems that the calculation of an exact maximum topological charge in the general PVL geometry is not possible.

5. Fractional plasmonic vortex

So far we have only considered the case in which all parameters j , l , m , and s are the integers. A question for the non-integer (or fractional) cases naturally arises. This section covers the case when those parameters have non-integer values and what would happen in that case. We also examine if the superposition rule ($j = l + m + s$) is still valid in the non-integer cases.

Non-integer l and m arise when the wavelength of the laser beam does not match that for the designed optical system. A spiral phase plate or an SLM induces a certain amount of phase delay and the amount of the modulated phase is intrinsically dependent on the wavelength of light. The parameters l and m are associated with the winding number of phase around the center axis, and thus they are integers when the modulated phase is an integer multiple of 2π . If the wavelength of incident light is different from the wavelength designed for the optical device, then the resultant amount of the modulated phase is not 2π anymore; it could go beyond or under 2π . For example, if the laser beam with the 600 nm wavelength illuminates the spiral phase plate or an SLM that induces 2π phase delay ($l = 1$) for the 660 nm, then the phase modulation would exceed 2π and the resultant helix winding number would be greater than one ($l \approx 1.1$). Likewise, the phase delay by the PVL is also dependent on the operating wavelength. If the PVL designed as $r = a + \lambda_{660}\phi / (2\pi)$ with the SP wavelength (λ_{660}) for free space wavelength of 660 nm ($m = 1$) is illuminated by a laser beam with a different wavelength of 600 nm, then the SP wave *feels* the distance λ_{660} more than 2π , which leads to a non-integer m ($1 < m < 2$).

In order to examine the effect of fractional l or m in the plasmonic vortex, we carried out numerical simulations. Let us first consider cases where either l or m is non-integer. If the helix winding number l is not an integer, the phase discontinuities are generated by the HVB along the axis and they cause a singular line linking the discontinuities in free space [29, 30]. As noted earlier, the plasmonic vortex pattern is superposition of SPs propagating in the PVL, and the phase of the SP wave is affected by the illuminating light phase. Consequently, those discontinuities also affect the vortex pattern and generate a singular line in the plasmonic vortex pattern. In the same manner as the fractional l , the PVL with fractional m generates a singular line. Recall that the PVL is designed by using equation $r = a + \lambda_{sp} \text{mod}(m\phi, 2\pi) / (2\pi)$. Because of different optical path lengths between $\phi = 0$ and $\phi = 2\pi$, the PVL with fractional m thus makes discontinuity along x -axis. This discontinuity makes a singular line and breaks the plasmonic vortex pattern as shown in Fig. 8(a) with $(l, m, s) = (0, 1.7, 0)$. In order to explain the effect of fractional l and m , phase profiles inside of vortex pattern are displayed in Figs. 8(c)-8(f). The phase profile is obtained along the circle whose center is coincident with center of vortex pattern as shown in Fig. 8(b), which

represents the phase of surface plasmon at the center of PVL. Figures 8(c) and 8(d) show the phase profiles in vortex patterns with $(l, m, s) = (0, 1.7, 0)$ and $(l, m, s) = (1.3, 0, 0)$, respectively. They show that the phase varies nonlinearly when m and l have fractional values.

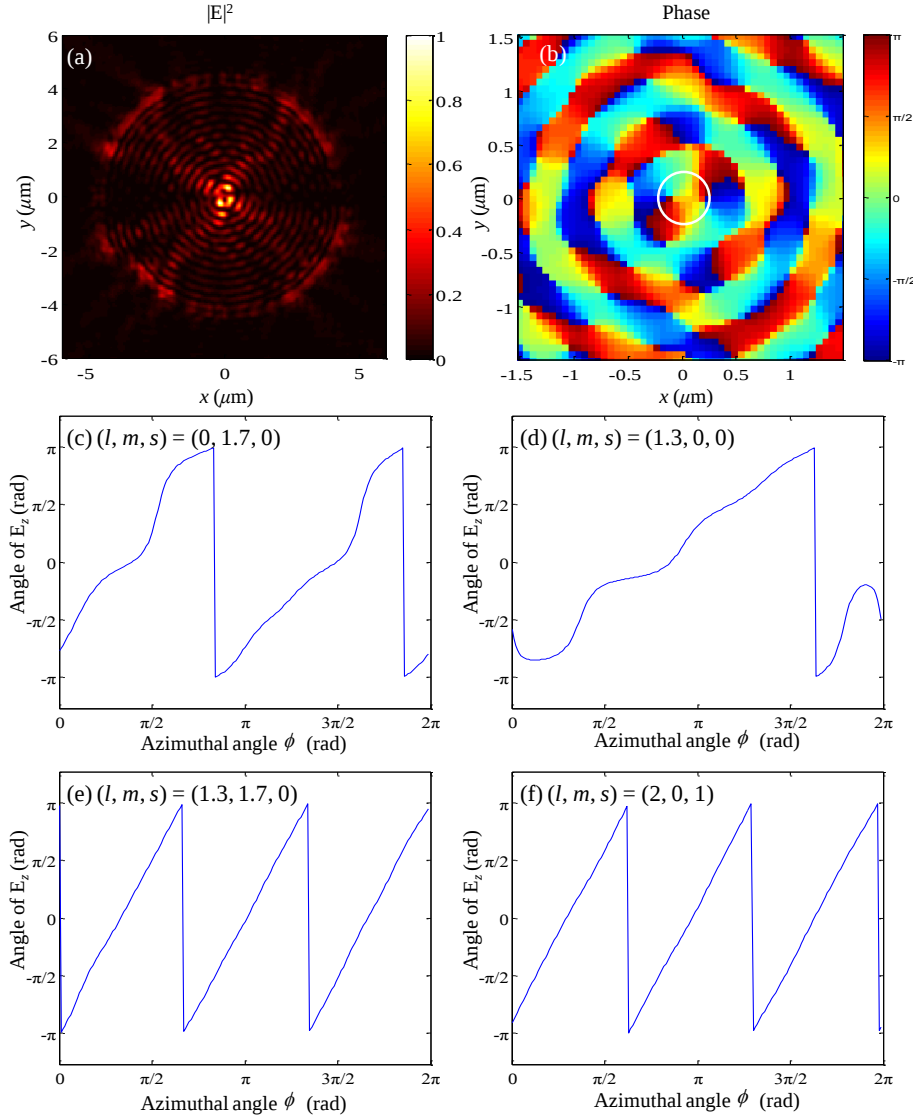


Fig. 8. Phase discontinuity breaks the plasmonic vortex pattern as shown in (a) with $(l, m, s) = (0, 1.7, 0)$. The phase of surface plasmon at the center of PVL is represented in (b) with $(l, m, s) = (0, 1.7, 0)$. The phase profile is obtained along the white circle and shown in (c) $(l, m, s) = (0, 1.7, 0)$, (d) $(l, m, s) = (1.3, 0, 0)$, (e) $(l, m, s) = (1.3, 1.7, 0)$, and (f) $(l, m, s) = (2, 0, 1)$. When j is fractional, the phase varies nonlinearly as shown in (c) and (d). When j is an integer, the phase varies linearly as shown in (e) and (f). This figure shows superposition property, $j = l + m + s$.

In the previous section, it was shown that the vortex topology j is determined by sum of l , m and s when each value is integer ($j = l + m + s$) and this property originates from the superposition of the phase profile. It is not trivial whether the aforementioned superposition rule is still valid in the non-integer case. To examine this issue, we carried out simulations in

which l and m are both non-integers whereas their sum $l + m$ is an integer. Figure 8(e) shows the phase profile in vortex pattern with $(l, m, s) = (1.3, 1.7, 0)$. Note that this can be regarded as a superposition of $(l, m, s) = (0, 1.7, 0)$ (Fig. 8(c)) and $(l, m, s) = (1.3, 0, 0)$ (Fig. 8(d)). The topological charge j of Fig. 8(e) is obtained as 3. It is noteworthy that its phase profile is the same as that of $(l, m, s) = (2, 0, 1)$ (Fig. 8(f)), whose topological charge j is 3.

So far we have covered the fractional parameters only for l and m . One may be inclined to ask about the effect of the fractional spin parameter s . By using the spin operator, the spin of the polarization is represented as $s = \sin(2\theta)\sin(\phi_{y-x})$, where the angle θ describes the angle between the amplitudes of the electric field components in the x and y directions, and ϕ_{y-x} is the relative phase difference of the electric field components in the x and y directions. In the case of the RCP, which is represented by $|R\rangle$, θ is $\pi/4$, ϕ_{y-x} is $\pi/2$, and $s = 1$. For the x -directional linear polarization ($|x\rangle$), θ is 0 and $s = 0$. In this equation, s is a real number between -1 and 1 when light has an elliptical polarization. Although the HVB has a fractional spin parameter s , light with a fractional s is unsuitable for PVL system. Because an elliptically polarized beam has different amplitudes between x and y directions, the intensity of excited SP waves is different with the position. Like the linearly polarized beam, the elliptically polarized HVB breaks uniformity of the SPs and it makes the asymmetric plasmonic vortex pattern. The radially polarized beam is represented by $|P_{\text{Radial}}\rangle = [\exp(-j\phi)|R\rangle + \exp(j\phi)|L\rangle] / 2^{1/2}$, where $|L\rangle$ stands for the LCP and ϕ is the azimuthal angle [17, 31]. This polarization is composed of the linear polarizations whose vector directions are toward the center of the beam. By using the spin operator, s of radial polarization is obtained as 0. Because the radially polarized beam excites the SP waves uniformly, it can be used in PVL system. Consequently, spin parameter s is one of only 1, 0 and -1 in the PVL system, and superposition property of PVL topology $j = l + m + s$ is satisfied at real number l , m and $s = 1, 0, -1$.

6. Conclusion

In conclusion, we proposed and analyzed the model for generating and manipulating a plasmonic vortex with PVL and HVB. The overall effects of spin polarization s , helix winding number l and the geometric charge of PVL m are explained. s and l are associated with the spin angular momentum and the orbital angular momentum, respectively. The topological charge j in the plasmonic vortex is given by the superposition rule ($j = l + m + s$), where l and m are given by real numbers and s is given by one of 1, 0, -1 . It is shown that the quantitative primary ring size of vortex pattern is one of the solutions of the Bessel function of the first kind and the primary ring sizes of simulation result coincide well with the solutions. We believe that our finding can pave a novel way to the generation and manipulation of plasmonic hot spots and vortices.

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