Uncertainty-managed phase-shifting digital holography

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Phase-shifting digital holography is a digital measurement technology of a complex optical field profile that uses focal plane array detectors without the loss of bandwidth. It has been known that the accuracy of phase-shifting digital holography is limited mainly by the phase tolerance of reference. In this Letter, it is revealed that the uncertainty in an expected signal is highly dependent on the phase of the signal itself, as well as the phase tolerance of the reference. Based on the uncertainty analysis, we propose a novel scheme of phase-shifting digital holography that exploits an uncertainty property to enhance the measurement accuracy even under significant reference phase tolerance. © 2012 Optical Society of America

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Phase-shifting digital holography is a technology used for profiling complex optical signals where several interferograms of signal and reference optical waves are digitally recorded by a focal plane array (FPA). Interferograms with different phase-shifts are analyzed to extract the amplitude and phase information of optical signal waves [1,2]. In the FPA, since the signal is measured and computed pixel by pixel in an independent and parallel manner, probable error in one pixel does not affect the estimation of the signal in the other pixels. This feature, whereby the error does not spread over to the other pixels is a unique advantage, which allows us to use the full bandwidth of the FPA of phase-shifting digital holography.

In principle, the signal in phase-shifting digital holography can be computed from two interferograms taken by two references with different phase shifts [3,4]. The signal and the reference at a given position on an FPA are represented, respectively, as the following complex numbers

$$X_S = |X_S| \exp(j\phi_S), \tag{1a}$$

$$X_{R,i} = |X_R| \exp(j\phi_{R,i}), \tag{1b}$$

where the subscript i means the i-th element among the predetermined set of phase shifts. The FPA measures the intensity of the interference of the signal and the i-th reference with a known phase shift. The acquisted interferogram is represented as

$$I_i = |X_S|^2 + |X_{R,i}|^2 + 2|X_S||X_{R,i}| \cos(\phi_S - \phi_{R,i}). \tag{2}$$

With two interferograms and using Eq. (2), we can calculate X_S. However, in practical measurement, because of finite tolerance of $\Delta\phi_{R,i}$ in reference phase $\phi_{R,i}$, the obtained X_S is just an expectation of true signal within finite uncertainty or confidence interval. Thus, an additional strategy for enhancing measurement accuracy is necessary.

The compensation techniques of the uncertainty involved in phase-shifting digital holography have been studied by many researchers. The least-square method was applied to analyze the errors [5]. Under an assumption that the phase-shifts of the references are the same over the whole interferogram, the average method [6] and the Max–Min algorithm [7] present reasonable and useful results. Thurman and Fienup suggested the sharpness matrix as a merit function to correct the phase errors [8]. The ratio of the twin image noise in the signal was applied to find unknown phase-shifts [9]. In practice, it is convenient to measure more than two interferograms to overcome the uncertainty in signal reconstruction.

In this Letter, an analysis on the uncertainty of phase-shifting digital holography is presented and a novel scheme of phase-shifting digital holography with reduced uncertainty is proposed. To describe the proposed algorithm intuitively, we use the graphical diagram shown in Fig. 1, which describes the process of estimating the signal with two interferograms (red and blue) in the complex plane, where a circle corresponds to an interferogram of Eq. (2). The radius of the circle corresponds to the measured intensity. The signal, X_S, can be estimated at one of the two distinct intersection points of the red and blue solid-lined circles and the other is the twin image noise, X_T. These intersection points are represented by the terms in the transformed coordinate system of Fig. 1(b) as

$$X_{\text{int}} = e^{j\theta} X'_{\text{int}} + (X^*_{R,1} + X^*_{R,2})/2, \tag{3}$$

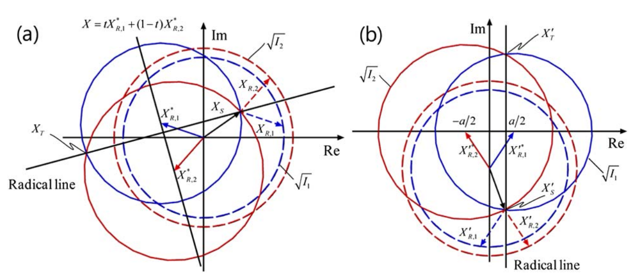

Fig. 1. (Color online) (a) Geometric interpretation of phase-shifting digital holography and (b) its expression in the transformed coordinate system.

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where the rotation angle θ is given by

$$\theta = \arg(X^*_{R,1} - X^*_{R,2}), \quad (4)$$

where $\arg(\cdot)$ is the argument function of a complex number and the radical line interconnecting X_S and X_T is mapped to a line parallel to the imaginary axis. The intersection points of two interferograms, I_1 and I_2 are obtained as

$$X'_{\mathrm{int}} = (I_2 - I_1)/(2a) \pm j\sqrt{2a^2(I_2 + I_1) - (I_2 - I_1)^2 - a^4}/(2a), \quad (5)$$

where a is the distance between $X'^*_{R,1}$ and $X'^*_{R,2}$, $a = |X'^*_{R,1} - X'^*_{R,2}|$. The signal X_S and twin image noise X_T are positioned symmetrically with respect to the line connecting the centers of the solid-lined circles in Fig. 1(a).

The necessary criterion of the reference for filtering twin image noise is established in the diagram shown in Fig. 2(a). Here, for simplicity of discussion, the phases of the references are set to $\phi_{R,1} = +\alpha$ and $\phi_{R,2} = -\alpha$, respectively, and the amplitude of the signal is supposed to be less than a fixed amplitude $|X_S|_{\max}$, which means that the signal is confined inside the region indicated by the dotted circle in Fig. 2(a). Considering the condition that all possible signals and twin image noise can be separated in the right-hand side and the left-hand side of the line of the centers, respectively, we can state a necessary condition for filtering the twin image noise as a criteria of the amplitude of the reference,

$$|X_R| \geq |X_S|_{\max} \sec \alpha, \quad (6)$$

where α is given by $\alpha = \arg(X_{R,2}X^*_{R,1})/2$. Therefore, the use of references with sufficient amplitude guarantees the reconstruction of the signal without the twin image noise.

Meanwhile, we focus on the uncertainty problem caused by the phase tolerance of reference. Let us prepare three references with $\phi_{R,1} = 60\ °$, $\phi_{R,2} = 180\ °$, and $\phi_{R,3} = 300\ °$ and corresponding interferograms, I_1, I_2, and I_3. When we know that the tolerance of the reference phase $\Delta\phi_{R,i}$ usually happens in practical experiments and induces some errors in interferogram, we can then try to change the reference within its given tolerance to find more accurate signal X_S. Figure 2(b) visualizes this interpretation process. Due to $\Delta\phi_{R,i}$, the circle of the expected signal from the single measurement becomes a circular shaped band, as shown in Fig. 2(b). The red-colored intersection area of the two circular-shaped bands is defined by the uncertainty region. The angular direction width and radial direction width of the uncertainty region are defined by the phase and signal uncertainties of the expected signal, respectively.

In Figs. 3(a) and 3(b), the phase and amplitude uncertainties of the signal extracted from two interferograms, I_2 and I_3, are plotted as a function of the phase of the expected signal in the polar form, where the radial position and the polar angle indicate the uncertainty and the phase of the expected signal, respectively. In the analysis, the amplitude of the reference is set to two-times larger than that of the signal, which is practically considerable as well as can satisfy the criteria of Eq. (6), and the phase tolerance is assumed to be 10°. It is seen that the uncertainties of the amplitude and phase of the signal are strongly dependent on the phase of the expected signal. The phase uncertainty is less than 4.5° except for the signals with ϕ_S in the ranges of 90 °–120° and 0°–30°. The amplitude uncertainty is, in particular, much smaller for the signals with ϕ_S in the ranges of 120°–180° and 300°–360° (the sectors indicated by dark color) than that for other signals. In Figs. 3(c) and 3(d), the phase and amplitude uncertainties of three-step phase-shifting digital holography using three interferograms I_1, I_2, and I_3 with the same tolerance of 10° are presented. The process of determining the signal in the three-step phase shifting holography is equivalent to finding the intersection point of three interferogram circles in the complex plane. However, due to the uncertainty, it is not plausible to find a single intersection point. We should determine an optimal point, minimizing the total error of the estimation. For simplicity, we can take the average of three intersection points of the probabilistic combinations of I_1, I_2, and

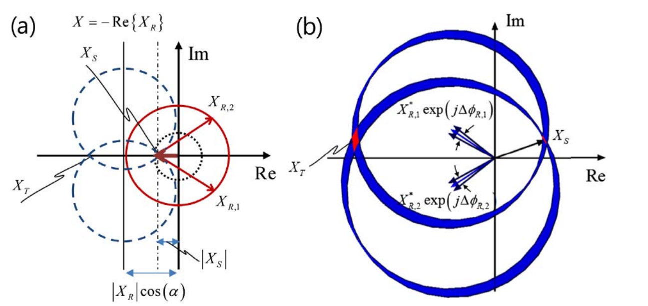

Fig. 2. (Color online) (a) Necessary condition of twin image noise elimination and (b) uncertainty region of the estimated signal which is represented as the red-colored intersection area of two blue circular-shaped bands.

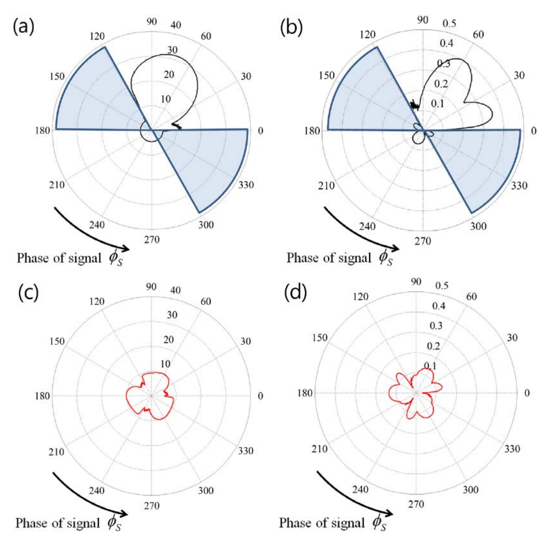

Fig. 3. (Color online) Uncertainties as a function of the phase of the expected signal for (a) phase and (b) amplitude of two-step holography, and (c) phase and (b) amplitude of three-step holography.

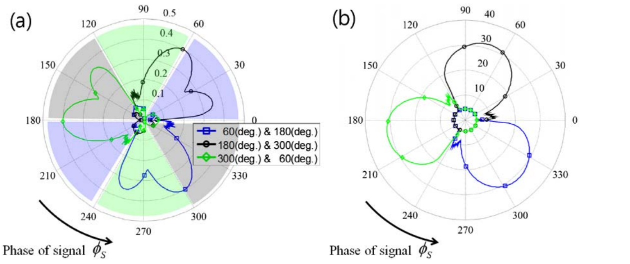

Fig. 4. (Color online) Minimum achievable uncertainties for (a) amplitude and (b) phase of signal in proposed method where tolerances of the reference phases are set to 10°.

I_3, $(X_{\text{int},1,2} + X_{\text{int},2,3} + X_{\text{int},3,1})/3$, where $X_{\text{int},1,2}$ ($X_{\text{int},2,3}$ and $X_{\text{int},3,1}$) denotes the intersection points of I_1 and I_2 (I_2 and I_3, and I_3 and I_1), respectively. As seen clearly in Fig. 3(c), the three-step phase shifting holography has a nonuniform phase uncertainty greater than the reference uncertainty of $\pm 5\,°$ and the amplitude uncertainty for some signals is considerable(>10%) as shown in Fig. 3(d). Comparing the results of Figs. 3(b) and 3(d), we can see that a two-step phase-shifting holography method is more accurate than a three-step phase-shifting holography in analyzing amplitude for the signals with ϕ_S in the ranges of $120 - 180\,°$ and $300 - 360\,°$.

From the observation of this uncertainty property of the two-step process, we devise an uncertainty-managed phase-shifting digital holography method. To explain this, we plot the amplitude and phase uncertainties for the combinatorial pairs of I_1, I_2, and I_3 on the same graph in Figs. 4(a) and 4(b), respectively, and find that each two-step process has its own range of signal phase that produces the smallest uncertainty. The complex plane is divided into six fan-shaped sectors, where a single two-step estimation of the signal is allowed to determine the signals in the interval where the estimation has its best (smallest) uncertainty. The signals in the blue, green and dark colored regions in Fig. 4(a) are to be estimated by using two corresponding interferograms. The overlap area represented by six small symmetric leaves around the center of Fig. 4(a) indicates the minimum amplitude uncertainty achievable with the proposed method. For example, the pair of I_1 and I_2 is used for estimating the signals expected to have the phase in the ranges of $0 \le \phi_S < 60\,°$ and $180 \le \phi_S < 240\,°$, and the pairs of I_2 and I_3, and I_3 and I_1 are used for their own regions. The maximum uncertainty of amplitude is less than 5% (0.05) in our analysis. For the same signals, we eliminate the shoulders of large phase uncertainty and establish the inherent limit of the phase uncertainty to a value of 4.5° as shown in Fig. 4(b). Comparing the uncertainties of the conventional method presented in Fig. 3 and those of the proposed method in Fig. 4, we conclude that the proposed method is advantageous for determining both amplitude and phase of signal with a practically reasonable reference intensity level.

The proposed algorithm provides the optimal strategy for reconstructing the signal with minimum uncertainty. This method can be extended straightforwardly to general schemes of phase-shifting digital holography using more than three references.

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