

Characteristics of complex light modulation through an amplitude-phase double-layer spatial light modulator

SUNGJAE PARK,¹ JINYOUNG ROH,¹ SOOBIN KIM,¹ JUSEONG PARK,² HOON KANG,² JOONKU HAHN,³ YOUNGJIN JEON,¹ SHINWOONG PARK,¹ AND HWI KIM^{1,*}

¹Department of Electronics and Information Engineering, College of Science and Technology, Korea University, Sejong-Campus, 2511 Sejong-ro, Sejong 30019, South Korea

²LG Display Co. Ltd., 245 LG-ro, Wollong-myeon, Paju-si, Gyeonggi-do, South Korea

³School of Electronics Engineering, Kyungpook National University, 80 Daehak-ro, Buk-Gu, Daegu 41566, South Korea

*hwikim@korea.ac.kr

Abstract: The complex modulation characteristics of a light field through an amplitude-phase double-layer spatial light modulator are analyzed based on the wave-optic numerical model, and the structural conditions for the optimal double-layer complex modulation structure are investigated. The relationships of interlayer distance, pixel size, and complex light modulation performance are analyzed. The main finding of this study is that the optimal interlayer distance for the double-layer structure can be found at the Talbot effect condition. For validating the practical usefulness of our findings, a high quality reconstruction of the complex computer-generated holograms and the robustness of the angular tolerance of the complex modulation at the Talbot interlayer distance are numerically demonstrated.

© 2017 Optical Society of America

OCIS codes: (090.2870) Holographic display; (070.6120) Spatial light modulators; (230.4170) Multilayers; (100.6890) Three-dimensional image processing.

References and links

1. J. Hong, Y. Kim, H.-J. Choi, J. Hahn, J.-H. Park, H. Kim, S.-W. Min, N. Chen, and B. Lee, "Three-dimensional display technologies of recent interest: principles, status, and issues [Invited]," *Appl. Opt.* **50**(34), H87–H115 (2011).
2. H. Kim, C.-Y. Hwang, K.-S. Kim, J. Roh, W. Moon, S. Kim, B.-R. Lee, S. Oh, and J. Hahn, "Anamorphic optical transformation of an amplitude spatial light modulator to a complex spatial light modulator with square pixels [invited]," *Appl. Opt.* **53**(27), G139–G146 (2014).
3. S. Choi, J. Roh, H. Song, G. Sung, J. An, W. Seo, K. Won, J. Ungnapatanin, M. Jung, Y. Yoon, H.-S. Lee, C.-H. Oh, J. Hahn, and H. Kim, "Modulation efficiency of double-phase hologram complex light modulation macro-pixels," *Opt. Express* **22**(18), 21460–21470 (2014).
4. H. Song, G. Sung, S. Choi, K. Won, H.-S. Lee, and H. Kim, "Optimal synthesis of double-phase computer generated holograms using a phase-only spatial light modulator with grating filter," *Opt. Express* **20**(28), 29844–29853 (2012).
5. S. Reichelt, R. Häussler, G. Fütterer, N. Leister, H. Kato, N. Usukura, and Y. Kanbayashi, "Full-range, complex spatial light modulator for real-time holography," *Opt. Lett.* **37**(11), 1955–1957 (2012).
6. J.-P. Liu, W.-Y. Hsieh, T.-C. Poon, and P. Tsang, "Complex Fresnel hologram display using a single SLM," *Appl. Opt.* **50**(34), H128–H135 (2011).
7. E. Ulusoy, L. Onural, and H. M. Ozaktas, "Full-complex amplitude modulation with binary spatial light modulators," *J. Opt. Soc. Am. A* **28**(11), 2310–2321 (2011).
8. P. Zhou and J. H. Burge, "Analysis of wavefront propagation using the Talbot effect," *Appl. Opt.* **49**(28), 5351–5359 (2010).
9. D. Im, E. Moon, Y. Park, D. Lee, J. Hahn, and H. Kim, "Phase-regularized polygon computer-generated holograms," *Opt. Lett.* **39**(12), 3642–3645 (2014).
10. J. An, G. Sung, S. Kim, H. Song, J. Seo, H. Kim, W. Seo, C.-S. Choi, E. Moon, H. Kim, H.-S. Lee, and U. Chung, "Binocular holographic display with pupil space division method," *SID Symposium Digest of Technical Papers* **46**, 522–525 (2015).
11. Y. Lim, K. Hong, H. Kim, H.-E. Kim, E.-Y. Chang, S. Lee, T. Kim, J. Nam, H.-G. Choo, J. Kim, and J. Hahn, "360-degree tabletop electronic holographic display," *Opt. Express* **24**(22), 24999–25009 (2016).

1. Introduction

The wave-optics feature of light is recognized as one of keys for a breakthrough in next generation display technology. The wave feature has been ignored, thought unworthy of discussion, or considered a hindrance, and so has been minimized in many conventional display systems. However, the wave-optics feature is of paramount importance in new generation display technology. A three-dimensional (3D) holographic display is considered to be the ultimate display architecture based on the wave-optical engineering of the light field [1]. A 3D holographic image is a synthetic result of interference and diffraction effects in volume space. For the high quality generation of a holographic image light field, effectively modulating the complex light field distribution on two-dimensional (2D) spatial light modulators (SLMs) is crucial. In practice, amplitude-only and phase-only SLMs are commercially available for light field modulation. However, the full complex modulation of both amplitude and phase of the light field is necessary for a holographic 3D display. There are several previous studies that emphasize the advantages and explain the physical mechanisms of complex light modulation [2–7]. One challenge is the construction of effective complex SLM architecture. A conventional approach is the optical interconnection of amplitude and phase SLMs. Two SLMs are optically connected by an optical relay system such as the conventional 4-f system. In this case, the pixel-to-pixel alignment is difficult and the system inevitably becomes bulky. To remove the alignment difficulty, macro pixel techniques were researched. One of the macro-pixel approaches is the double-phase hologram method. Two phase-only pixels are interfered coherently and the interference of the range phase modulated optical wave then results in a full-range complex modulation [3, 4]. The macro-pixel structure based on the double-phase hologram technique is advantageous in being free of misalignment and interlayer diffraction problems, however, a highly developed fabrication technique for ultra-high resolution SLM panel is required. Direct stacking of the amplitude-only and phase-only SLMs-the amplitude-phase double-layer structure is a basic idea and seems to be advantageous in terms of its simplicity in structure design and fabrication. However, it is impossible to reduce the interlayer distance between the amplitude and phase SLMs to zero; thus, the interlayer distance induces interlayer diffraction and cross-talk contamination is expected, which prevents the perfect pixel-to-pixel matching interconnection of two SLMs. This has been accepted as a serious problem, and it is understood to be a practical limitation of double-layer complex SLM structure.

In this paper, we study the interconnection problem in the amplitude-phase double-layer structure from a novel viewpoint. In the proposed approach, the black matrix (BM) pattern of the SLM is important, because it induces a complicated interlayer diffraction pattern and breaks the common sense of monotonic degradation in complex modulation by increasing the interlayer distance. The analysis focuses on the variation of the interconnection performance with a change in interlayer distance. Our main finding is the important role of the Talbot effect, which is ascribed to the BM pattern [8]. The BM-patterned amplitude SLM is replicated at the Talbot position. The Talbot positions are multiples of unit Talbot distance. This opens the possibility that we can discover a way to construct a double-layer complex SLM structure with an optimal interlayer distance that can be adjusted under practical fabrication conditions. Our analysis also shows that the deviation of the practical complex modulation from the ideal complex modulation with zero interlayer distance can be minimized at the Talbot interlayer distances. The most powerful application of a complex SLM is the holographic 3D display [2]. In the context of a holographic 3D display, the relation of complex modulation and the image quality of computer-generated holograms (CGHs) is analyzed. The correlation of the image formation quality and the Talbot distance are the focus of the analysis. Additionally, the angular tolerance at those optimal Talbot interlayer distances for the observation direction is analyzed, and a comment for the angular tolerance robustness is derived.

In section 2, a wave-optics model of double-layer complex SLM architecture is developed. In section 3, the Talbot interlayer distance of the amplitude and phase SLMs is analyzed. While in section 4, the angular tolerance of the CGH formation produced by the double-layer SLM is analyzed. The concluding remarks are given in section 5.

2. Wave-optics model of amplitude-phase double-layer complex SLM

In this section, the wave-optics model of an amplitude-phase double-layer complex SLM is developed. Figure 1 shows the schematic of a holographic display system with a double-layer complex modulation SLM. Displaying a holographic three-dimensional (3D) image, a letter 'L' floating in free space for the single eye of an observer, is illustrated in Fig. 1. The schematic of the double-layer complex SLM shown in Fig. 1(a) can be split into the first part of a complex SLM and the second part of the holographic display system. The first free-space interconnection section from the amplitude-only SLM plane ($x'_1 y'_1$ plane) to the phase-only SLM plane ($x_1 y_1$ plane) is coplanar with the virtual complex SLM plane ($x_1 y_1$ plane). The interlayer distance between the amplitude-only SLM and the phase-only SLM is denoted by d . The second observation section is from the virtual complex SLM plane ($x_1 y_1$ plane) to the retinal plane ($x_2 y_2$ plane). The input CGH pattern for the holographic image light field generation is formed on the virtual SLM plane indicated by the red box in Fig. 1. The CGH pattern is the light field distribution produced by the modulation of an incident light wave through the double-layer SLM. The double-layer SLM is composed of a rear amplitude-only SLM and a front phase-only SLM. Thus, the modulation transmittance of the rear amplitude-only SLM is represented by $A(x'_1, y'_1)$, $0 \leq A(x'_1, y'_1) \leq 1$, while that of the former phase SLM is given by $\exp(j\phi(x_1, y_1))$, $0 \leq \phi(x_1, y_1) \leq 2\pi$. The incident light field is sequentially transmitted through the first rear amplitude-only SLM and the second front phase-only SLM layers and produces the complex CGH pattern at the virtual SLM plane via the form of light field distribution $G(x_1, y_1)$.

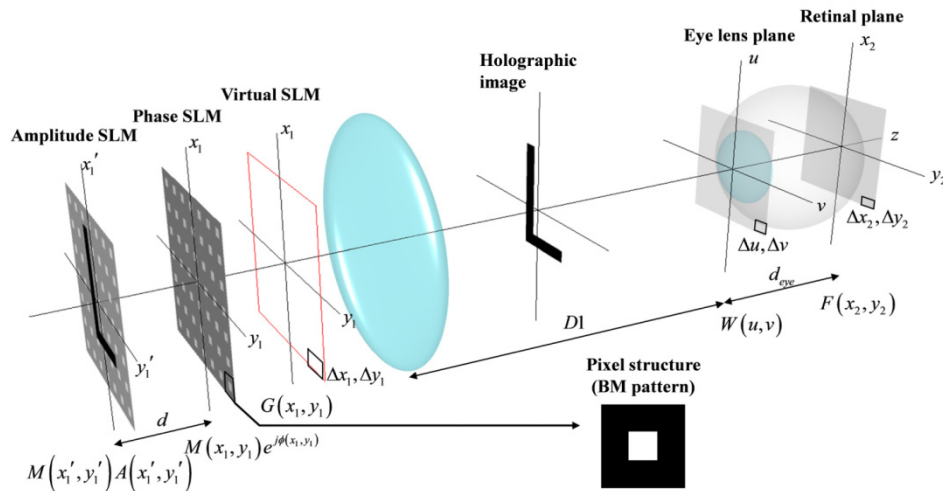


Fig. 1. Numerical model of the holographic 3D display system with an amplitude-phase double-layer complex spatial light modulator

The part of the holographic display system is composed of the virtual SLM, a field lens with the focal length D_1 , and the observer's eye. Here, the eye is modeled by the eye lens and

retina planes [9]. The eye lens plane is supposed to be at the focal plane of the field lens, D_1 distant from the field lens. The optical field distributions on the eye lens and retinal planes are denoted by $W(u, v)$ and $F(x_2, y_2)$, respectively. In order to synthesize a perfect complex holographic image in free space, a full complex light field distribution should be formed in the virtual SLM plane [9] and should be continuously profiled in both amplitude and phase. However, in practice, the realizable light field in the virtual SLM plane is constrained by the structural factors of the double-layer SLM such as pixel size, BM pattern, and interlayer distance. The finite interlayer distance between two SLMs and the finite fill factor of the pixel have a significant influence on light transmission characteristics. The resultant finite-distant double-layer BM-patterned SLM structure is expected to produce a variety of interesting features in complex modulations and CGH reconstruction.

The objective of the spatial light modulation system is the construction of an ideal complex light field at the virtual SLM plane. We take a simple modulation strategy such that the amplitude and phase SLMs have the modulation profiles of the amplitude and phase profiles of the target CGH patterns, given, respectively, by

$$A(x'_1, y'_1) = |CGH(x'_1, y'_1)|, \quad (1)$$

$$\phi(x_1, y_1) = \text{Arg}[CGH(x_1, y_1)]. \quad (2)$$

However, as mentioned, the actually obtained field distribution at the virtual SLM plane is influenced by the BM patterns and interlayer distance, which is represented by

$$G(x_1, y_1) = \{[M(x'_1, y'_1)A(x'_1, y'_1)] \otimes h_{gap}(x_1, y_1)\} \times \{M(x_1, y_1) \exp[j\phi(x_1, y_1)]\}, \quad (3)$$

where $h_{gap}(x, y)$ and $M(x, y)$ are the interlayer distance transfer function and the binary BM transmittance function. $\otimes$ is the convolution operator. Here, the BM patterns for the amplitude and phase SLMs are supposed to be the same. The transmittance function of the BM is defined as

$$M(x, y) = \sum_{m,n} \text{rect}\left(\frac{x - ma}{f_B a}\right) \text{rect}\left(\frac{y - na}{f_B a}\right), \quad (4)$$

where a and f_B are the pixel pitch and the fill factor parameter, respectively, and $(x, y) = (x_1, y_1)$ or $(x, y) = (x'_1, y'_1)$. The fill factor of the rectangular aperture is defined by f_B^2 , where $0 < f_B^2 \leq 1$. The BM pattern of the phase SLM casts a shadow by blocking the optical field passing through the first amplitude SLM panel.

An optical wave modulated by the amplitude SLM, $A(x'_1, y'_1)$, is propagated toward the phase SLM and passes through the BM pattern represented by the binary amplitude mask function $M(x_1, y_1)$. The optical field $G(x_1, y_1)$, obscured by the BM mask, continues to propagate to the retinal plane, where the optical field distribution, $F(x_2, y_2)$, is expected to contain the diffraction pattern of the BM mask pattern. Assuming the ideal condition that the interlayer distance is 0, we can set the transfer function $h_{gap}(x_1, y_1)$ to $h_{gap}(x_1, y_1) = \delta(x_1, y_1)$ and $(x'_1, y'_1) = (x_1, y_1)$, and the complex transmittance is nearly perfect and represented by

$$G_{opt}(x_1, y_1) = M(x_1, y_1) \times A(x_1, y_1) \exp[j\phi(x_1, y_1)]. \quad (5)$$

In wave-optics modeling, the direct calculation of Eq. (3) is difficult since the interlayer distance is relatively short and the direct application of the Fresnel diffraction theory for the calculation of $h_{gap}(x_1, y_1)$ under the paraxial approximation is inappropriate. We pose an appropriate wave-optics model and a wave propagation algorithm for the system depicted in Fig. 1. Before developing the practical algorithm of the inter-layer wave propagation for Eq. (3), we need to describe an essential element of the theory, the generalized cascaded Fresnel transform [9]. It can be said that the main wave optic algorithms of this paper are (i) the cascaded Fresnel transform describing the relation of $G(x_1, y_1)$ and $F(x_2, y_2)$, and (ii) the inter-layer wave propagation algorithm of determining $G(x_1, y_1)$ of Eq. (3).

The generalized cascaded Fresnel transform describes the mathematical relationship between the optical field at the virtual SLM, $G(x_1, y_1)$, and that at the retinal space of the observer, $F(x_2, y_2)$, as shown in Fig. 1, which is segmented into forward and inverse generalized cascaded Fresnel transforms, CdFr and ICdFr, given by

$$F(x_2, y_2) = \text{CdFr}\{G(x_1, y_1); D_1, d_{eye}, f_{eye}, R_{eye}\} \text{ and}, \quad (6)$$

$$G(x_1, y_1) = \text{ICdFr}\{F(x_2, y_2); D_1, d_{eye}, f_{eye}\}, \quad (7)$$

where the eye-lens with a finite aperture is inserted in the intermediate plane between the virtual SLM and the retinal plane. R_{eye} is the radius of the eye lens aperture. The cascaded Fresnel transform $F(x_2, y_2) = \text{CdFr}\{G(x_1, y_1)\}$ is composed of the two elementary transforms FrT_1 and FrT_2 defined, respectively, as

$$W(u, v) = FrT_1\{G(x_1, y_1)\} \text{ and}, \quad (8)$$

$$F(x_2, y_2) = FrT_2\{t(u, v)W(u, v)\}. \quad (9)$$

The first integral transform, $W(u, v) = FrT_1\{G(x_1, y_1)\}$, is represented by

$$W(u, v) = \frac{e^{jkF}}{j\lambda D_1} \iint G(x_1, y_1) e^{-j\frac{2\pi}{\lambda D_1}(x_1 u + y_1 v)} dx_1 dy_1. \quad (10)$$

$W(u, v)$ is the optical field distribution on the eye lens plane (u-v plane). The second integral transform, $F(x_2, y_2) = FrT_2\{t(u, v)W(u, v)\}$ is represented by

$$F(x_2, y_2) = \frac{e^{j\frac{\pi}{\lambda d_{eye}}(x_2^2 + y_2^2)}}{j\lambda d_{eye}} \iint t(u, v) W(u, v) e^{-j\frac{2\pi}{\lambda d_{eye}}(ux_2 + vy_2)} dudv, \quad (11)$$

where the transmittance function $t(u, v)$

$$t(u, v) = e^{j\frac{\pi}{\lambda} \left(\frac{1}{D_1} + \frac{1}{d_{eye}} - \frac{1}{f_{eye}} \right) (u^2 + v^2)} \text{circ}\left(\left[u^2 + v^2\right] / \rho^2\right), \quad (12)$$

and ρ is the eye pupil radius. The inverse transforms of FrT_1 and FrT_2 , $G(x_1, y_1) = \text{IFrT}_1\{W(u, v)\}$ and $W(u, v) = t^{-1}(u, v) \text{IFrT}_2\{F(x_2, y_2)\}$ can be

straightforwardly derived [9]. In a numerical implementation, the sampling scheme is important. Let the x and y-directional sampling intervals of the virtual SLM be represented by Δx_1 and Δy_1 , respectively. Let the number of sampling intervals in the x and y-axis be N . The sampling intervals, Δu and Δv , in the eye lens plane are then determined in the fast Fourier transform (FFT) based computation scheme by

$$(\Delta u, \Delta v) = (\lambda D_1 / (N \Delta x_1), \lambda D_1 / (N \Delta y_1)). \quad (13)$$

Similarly, the sampling intervals in the output plane are $\Delta x_2 = \lambda d_{eye} / (N \Delta u)$ and $\Delta y_2 = \lambda d_{eye} / (N \Delta v)$. The output plane sampling intervals, Δx_2 and Δy_2 , are solved for the input plane sampling intervals, Δx_1 and Δy_1 , as

$$(\Delta x_2, \Delta y_2) = (\Delta x_1 d_{eye} / D_1, \Delta y_1 d_{eye} / D_1). \quad (14)$$

In addition, based on the above mathematical model, the mathematical model of the oblique observation can be derived. The oblique observation can be thought to be modeled by the lateral shift of the eye lens as depicted in Fig. 2. Under the Fresnel transform scheme, the lateral shift of the eye lens aperture is equivalent to the phase shift of the light field at the virtual SLM plane and that of the light field at the retinal plane. Therefore, the oblique observation is modeled such that the observation eye is shifted laterally by h . The optical field with a linear phase shift at the virtual SLM plane is then delivered to the spatially shifted eye pupil and the optical wave is sequentially delivered to the retina plane, where the resultant optical field also has a linear phase shift. This picture is mathematically expressed as follows. The relationship between the lateral shift of the eye pupil, from $W(u, v)$ to $W(u-h, v)$, and the linear phase shift terms at both the virtual SLM plane (x_1, y_1) and the retina plane (x_2, y_2) is analyzed. The spatially shifted eye pupil $W(u-h, v)$ along the u-axis produces the phase shifted optical fields distribution at both SLM and retinal planes, respectively, as

$$IFrT_1 \{t(u-h, v)W(u-h, v)\} = G(x_1, y_1) \exp\left(j2\pi h \frac{x_1}{\lambda d_1}\right) \text{ and}, \quad (15)$$

$$FrT_2 \{t(u-h, v)W(u-h, v)\} = F(x_2, y_2) \exp\left(-j2\pi \frac{h}{\lambda d_2} x_2\right). \quad (16)$$

It is seen that, for the oblique observation, the optical field distributions at the virtual SLM and the retina plane are manipulated with the carrier wave multiplication denoted above.

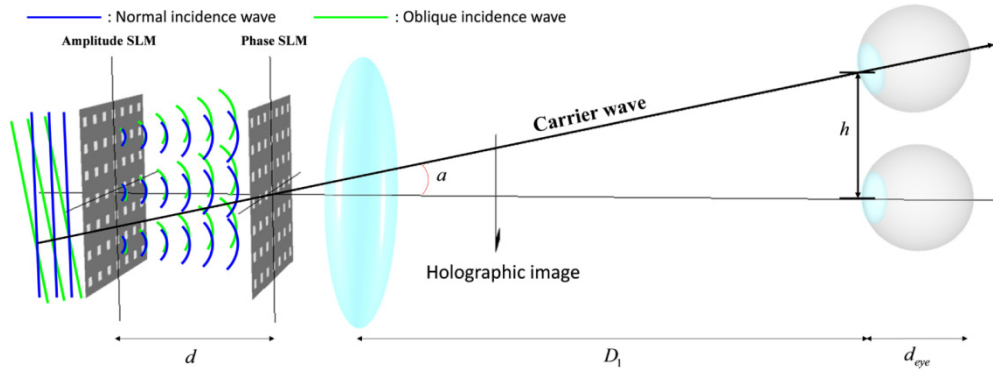


Fig. 2. Numerical model of an oblique observation of the amplitude-phase double-layer BM patterned complex spatial light modulator

The numerical inter-layer wave propagation algorithm of solving Eq. (3), that is, determining $G(x_1, y_1)$, is based on the use of cascaded Fresnel transform pair. In the first step, the eye focus is adjusted to the amplitude SLM plane, then the optical field at the retinal plane is set to $M(-x_2(D_1+d)/d_{eye}, -y_2(D_1+d)/d_{eye})A(-x_2(D_1+d)/d_{eye}, -y_2(D_1+d)/d_{eye})$, which is a contraction mapping of the complex optical field at the amplitude SLM plane. The finite interlayer distance d can give perspective effect in the observation. But, under the condition, $d \ll D_1$, the perspective difference of the amplitude and phase SLMs is ignorable, so the use of $M(-x_2D_1/d_{eye}, -y_2D_1/d_{eye})A(-x_2D_1/d_{eye}, -y_2D_1/d_{eye})$ for the contracted image at the retina plane is reasonable. At this stage, the eye lens focal length f_{eye} should be adjusted to $f_{eye} = 1/[1/(F-d) + 1/d_{eye}]$. The optical field backside the phase SLM plane is calculated by the transfer of the optical field at the retina plane to the phase SLM plane through the inverse cascaded Fresnel transform,

$$G_A(x_1, y_1) = ICdFr\{M(-x_2D_1/d_{eye}, -y_2D_1/d_{eye})A(-x_2D_1/d_{eye}, -y_2D_1/d_{eye}); D_1, d_{eye}, f_{eye}\}. \quad (17)$$

Next, the optical field after passing through the phase SLM is obtained as

$$G(x_1, y_1) = G_A(x_1, y_1) \times \exp(j\phi(x_1, y_1)) \times M(x_1, y_1) \quad (18)$$

With the above observation process be denoted by $Trcas$, the wave propagation algorithm $Trcas$ is the paraxial approximation of Eq. (3). This result can be described as

$$G(x_1, y_1; d) = Trcas\{M(x'_1, y'_1)A(x'_1, y'_1), M(x_1, y_1)\exp(j\phi(x_1, y_1)); d, M\} \\ \square \{[M(x'_1, y'_1)A(x'_1, y'_1)] \otimes h_{gap}(x_1, y_1)\} \times \{M(x_1, y_1)\exp[j\phi(x_1, y_1)]\}. \quad (19)$$

For the observation simulation, the generalized cascaded Fresnel transform of Eqs. (6) and (7) is used. By changing the eye focus f_{eye} , the observer can see the accommodation effect on the double-layer complex SLM as well as on the complex computer-generated hologram images. Of course, the algorithm described above can be applied to the case of oblique observation condition Eqs. (15) and (16). The simulation results for the verification of the developed optical model are presented in Fig. 3. In the simulation, the interlayer distance d and the observation distance D_1 are set to 0.05 and 1m, respectively.

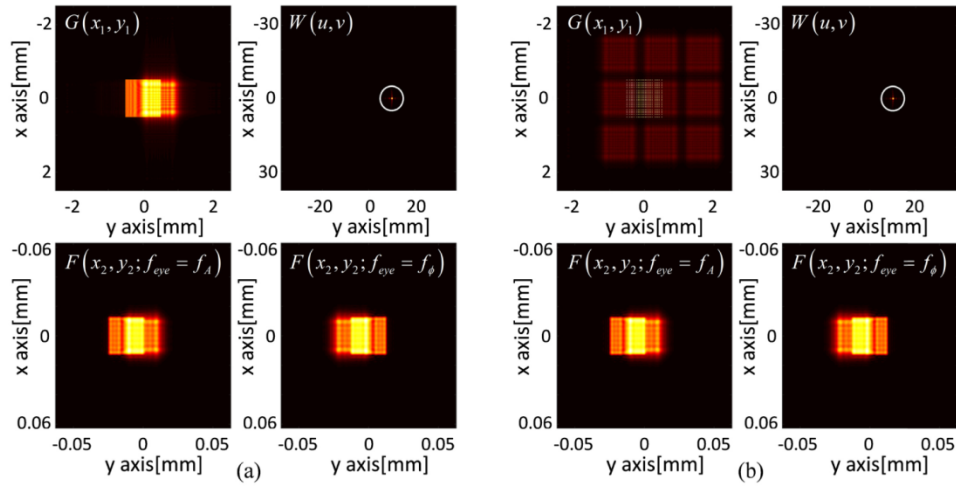


Fig. 3. Oblique observation of the double-layer SLM with finite interlayer distance; (a) observation simulation results of the double layer without BM patterning and (b) with BM patterning. The upper left and upper right figures of (a) and (b) are the optical field distributions at the virtual SLM plane and the eye lens plane, $G(x_1, y_1)$ and $W(u, v)$, respectively. The lower left and right figures are the observation images for the focus adjusted to the rear ($f_{eye} = f_A$) and front ($f_{eye} = f_\phi$) layers.

In Figs. 3(a) and 3(b), the simulation of an oblique observation of double layer structure with and without the BM patterns of Eq. (4) are shown. In the simulation, the wavelength is set to 633nm and the rectangular BM pattern with 11% fill factor, f_B^2 , is assumed as shown in Fig. 1. The simulation results demonstrate (i) the relative lateral shift of the rear and front SLM layers and the accommodation effects indicating that the focus points can be dynamically adjusted. In the case of the observation of a BM-patterned double layer SLM, the optical field distribution at the virtual SLM is quite interesting due to the replicative diffraction pattern of the BM pattern. The optical field delivered through the eye pupil as shown in the upper right figure in Fig. 3(b) is filtered out by the finite pupil aperture; the observed optical field distribution at the retinal plane created is similar to those of the first simulation of a non-BM-patterned SLM layer structure. Through a series of numerical simulation tests, the validity and extensibility of the constructed numerical model have been guaranteed. From the next sections, the investigation of the complex modulation characteristics of the amplitude and phase SLMs is performed based on the constructed numerical model.

3. Talbot interlayer distance of the double-layer complex SLM

Employing the wave-optics model, we investigate the structural conditions for optimal complex modulation. Assuming we want to make a simple hologram image ‘L’ in the intermediate plane ($z = z_{center}$), the CGH pattern is obtained by the inverse cascaded Fresnel transform of ‘L’. Let the object image be denoted by $I(x_2, y_2)$ and place it on the retina plane. The CGH pattern is then obtained by the virtual SLM plane as

$$CGH(x_1, y_1) = ICdFr\{I(x_2, y_2); D_1, d_{eye}, f_{eye}\}, \quad (20)$$

where the eye is supposed to focus on the holographic image ‘L’, and the focal length of the eye f_{eye} is then adjusted to

$$f_{eye} = 1 / \left[1 / (D_1 - z_{center}) + 1 / d_{eye} \right]. \quad (21)$$

For the CGH, the optical field distribution at the virtual SLM plane is obtained by Eq. (19). For investigating the influence of the interlayer distance d , we take the functions $E_1(d)$ and $E_2(d)$, indicating the complex modulation performance at the virtual SLM plane and the retinal plane, respectively. The complex field $G(x_1, y_1; d=0)$ is taken as the reference field $G_{ref}(x_1, y_1)$, and the deviation between $G_{ref}(x_1, y_1)$ and $G(x_1, y_1; d)$ is measured as a function of d . The error function is defined by

$$E_1(d) = \left\| G_{ref}(x_1, y_1) - G(x_1, y_1; d) \right\|_2. \quad (22)$$

The first measure $E_1(d)$ indicates how much the interlayer diffraction influences the deviation with an increase of the interlayer distance d . The second measure is designed as the deviation of the observed image from the reference observation image at the retina plane. The reference observation field is defined by $F_{ref}(x_2, y_2)$ generated by $G_{ref}(x_1, y_1)$ with the interlayer distance $d=0$,

$$F_{ref}(x_2, y_2) = CdFr\{G_{ref}(x_1, y_1); D_1, d_{eye}, f_c, R_{eye}\}. \quad (23)$$

The observation field $F(x_2, y_2; d)$ is generated by $G(x_1, y_1; d)$ with the non-zero interlayer distance

$$F(x_2, y_2; d) = CdFr\{G(x_1, y_1; d); D_1, d_{eye}, f_c, R_{eye}\}. \quad (24)$$

The second deviation measure at the retina plane is defined by

$$E_2(d) = \left\| F_{ref}(x_2, y_2) - F(x_2, y_2; d) \right\|_2. \quad (25)$$

$E_2(d)$ is the measure of the SNR of the observed image, i.e., optical field at the retina plane. It is expected that the two measures are positively correlated. In addition, the relative diffraction efficiency (DE) at the retina plane is defined as the energy ratio focused on the signal region,

$$DE(d) = \frac{\iint \Gamma(x_2, y_2) |F(x_2, y_2; d)|^2 dx_2 dy_2}{\iint \Gamma(x_2, y_2) |F_{ref}(x_2, y_2)|^2 dx_2 dy_2} \times \frac{\iint |F_{ref}(x_2, y_2)|^2 dx_2 dy_2}{\iint |F(x_2, y_2; d)|^2 dx_2 dy_2}, \quad (26)$$

where $\Gamma(x_2, y_2)$ is the indicator of the signal area. The DE is the relative measure, not for estimating absolute energy distribution but for reflecting the relative signal formation quality without consideration about the absolute energy efficiency. While, $E_1(d)$ and $E_2(d)$ measures are absolute measures evaluating the absolute difference from the reference signal.

In the numerical analysis, the pixel size and the number of sampling points are taken as $a = 10\mu\text{m}$ and $N_x = N_y = 1025$. The focal length of the field lens and the object position are $f = 1\text{m}$ and $z = 0.2\text{m}$. The wavelength used in the simulation is 633nm. For analysis convenience, we take a two-dimensional (2D) letter 'L' as the holographic objective image, as shown in Fig. 1. With a finite interlayer distance, the complex modulation deviates from the perfect complex modulation profile $G_{ref}(x_1, y_1)$; a monotonically increasing deviation

would be a natural expectation. However, as shown in Figs. 4(a), our numerical simulation shows that the optimal interlayer distance is periodically obtained at the Talbot position, where the Talbot position is given by

$$z_{T,m} = \gamma 2a^2 / \lambda, \quad (27)$$

where γ , a , and λ are the order of the Talbot position, SLM pixel pitch, and wavelength. Moreover, through the comparison of $E_1(d)$ and $E_2(d)$, the analysis shows that the Talbot position is matched to the minimum deviation points and the maximum SNR position as presented in Figs. 4(b). Moreover, the observed images at the retina plane shown in Fig. 4(c) are the holographic field $F(x_2, y_2; d)$ for the selected positions (1)-(5) indicated in Fig. 4(a).

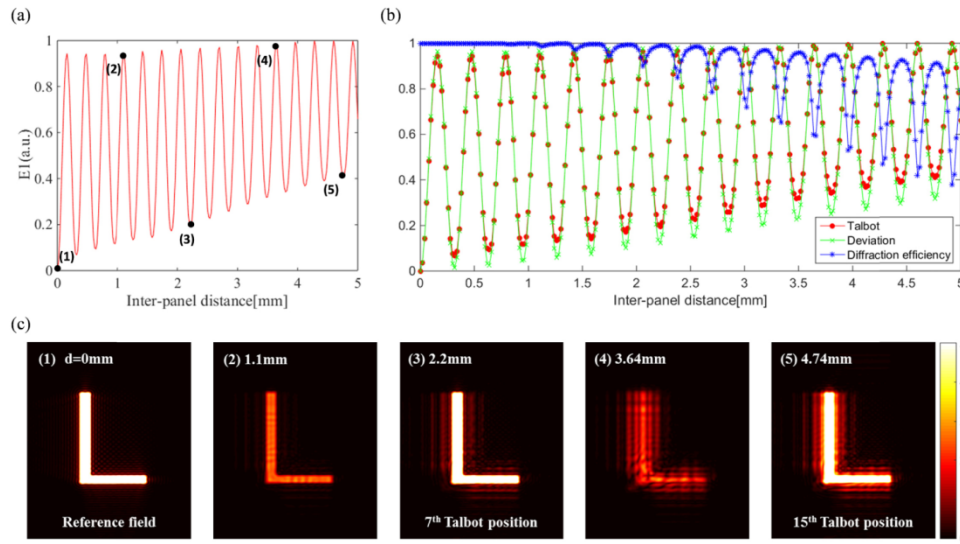


Fig. 4. Talbot effect analysis results. (a) $E_1(d)$ measure and (b) $E_2(d)$ and $DE(d)$ measures with a comparison of $E_1(d)$ for pixel size $10\mu\text{m}$, and (c) the observation image at the retina plane $F(x_2, y_2; d)$ in the selected positions indicated in (a).

The quality of observed image shows oscillatory behavior with respect to the inter-panel distance. The observed images at the positions (2) and (5) of distance 2.2mm and 4.74mm are closer to the reference image ($d = 0\text{mm}$) than those of the positions (2) and (4) of shorter distance 1.1mm and 3.64mm in terms of energy efficiency and image quality, respectively. Therefore, it is argued that the free space interconnection of the amplitude and phase SLMs can be optimized at the Talbot position. The Talbot position tends to be smaller for the small pixel size, but we can choose a practical Talbot position from several higher-order Talbot positions to engineer the architecture of the double-layer SLM. Also, it is expected that as the pixel size decreases further, the higher order Talbot position becomes more practical spot for the constitution of double-layer SLM. The contrast between the image quality and energy efficiency of Talbot and non-Talbot position images is quite clear even around the higher-order Talbot positions.

4. Angular tolerance of complex modulation

Due to the finite interlayer distance, a pixel-to-pixel alignment mismatch occurs, which invokes the holographic image degradation when the observer views the holographic panel

from an oblique direction. As shown in Fig. 2, the observation direction is influential in holographic image reconstruction. Under oblique observation, the observer sees the occlusion of the rear amplitude SLM panel via the front phase SLM pane; the occlusion changes with the observation direction. This alignment mismatch leads to considerable quality degradation of hologram images. Figures 5(a) and 5(b) present the magnified parts of the complex field at the virtual SLM plane for the spatial shifts of the eye, $h = 16.7\text{mm}$ and $h = 10.6\text{mm}$. In the case of $h = 16.7\text{mm}$, the active area of the amplitude SLM panel is relatively well matched to that of the phase SLM panel, resulting in a high transmission of almost all pixels and a high quality hologram image (lower right square in Fig. 5(a)), while in the case of $h = 10.6\text{mm}$, the BM part of the rear amplitude SLM is matched to the active area of the phase SLM, resulting in non-uniform low transmission and a poor quality hologram image (lower right square in Fig. 5(b)). For the observation angle where the black part of the BM pattern of the rear amplitude SLM overlaps with the pixel of the front phase SLM, the holographic 3D image is abruptly degraded.

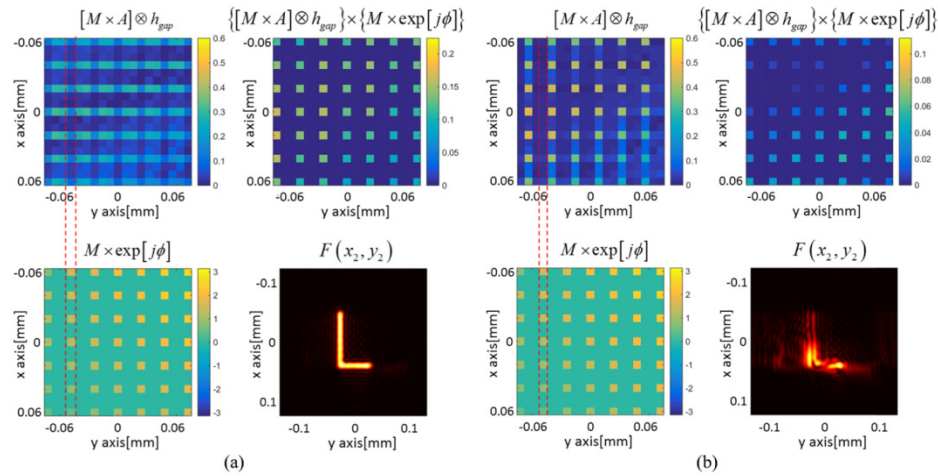


Fig. 5. Magnified view of the complex field distributions at the virtual SLM plane, for the spatial shifts of the eye; (a) $h = 16.7\text{mm}$ and (b) $h = 10.6\text{mm}$.

In Fig. 6, the behavior of the DE functions at the first four Talbot interlayer distances is drawn with the observation angle. The fluctuation pattern in the DE caused by the occlusion of the BM patterns is changed by the order of Talbot distance. At the lowest order of Talbot distance ($d = 0.316\text{mm}$), the first dip appears at the observation angle of 0.6° . In terms of DE, the dip indicates the angular observation position where an observer can see the image quality degradation clearly. The angular interval of the image degradation decreases as the order of the Talbot position increases. The angular width of the dips are very narrow, thus, in practice, adjusting the observation position to the outside region of the dips is recommended. Comparison of the cases in Fig. 6 demonstrates that the angular interval between the dips for the fundamental Talbot position are longest; thus, the first Talbot position is the best interlayer distance even in terms of angular tolerance. Outside the dips, the DE values are seen to be almost reserved. In particular, this property of the angular tolerance would allow us to use the double-layer SLM structure for binocular holographic 3D displays [10, 11].

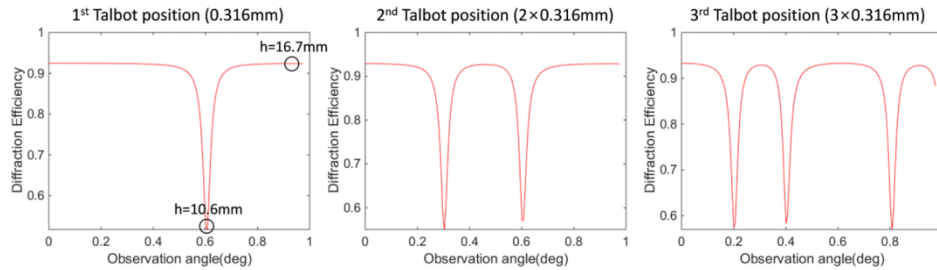


Fig. 6. $DE(d)$ measures at the first three Talbot interlayer distances with change in the observation angle.

5. Conclusion

In conclusion, the complex modulation characteristics of the double-layer SLM structure have been investigated. The analysis has shown that the interlayer distance between the amplitude and phase SLM layers for optimal complex modulation should be determined at the Talbot effect positions. The Talbot distance allows practical construction of a double-layer complex SLM structure. The oblique observation is critical for binocular holographic 3D display architecture. In terms of angular tolerance of the holographic image quality, the optimal interlayer distance is thought to be the fundamental Talbot position. It was revealed that the amplitude-phase double-layer SLM structure is a promising complex modulation SLM architecture since, at the Talbot distance, the attachment of amplitude and phase SLMs with high quality complex field generation is possible. The realistic possibility of the application of the double-layer SLM for practical holographic 3D displays can be expected.

Acknowledgment

This research was supported by a Korea University grant.