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Regularized Huygens' plasmonic wave field synthesis using a metal-clad plasmonic waveguide array

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Huygens' principle states that point sources are the basis of optical wave field generation, and an array of point sources with complex amplitudes that are separated by subwavelength distances can generate a desired optical field distribution. In field synthesis based on the Huygens' principle, the construction of ideal point sources has been overlooked when compared to other elements in optical field synthesis engineering, such as complex modulation. However, the construction of ideal point sources should be considered an important goal because the use of non-ideal point sources generates considerable optical noise in the background of the synthesized field distribution. In this Letter, we investigate Huygens' plasmonic wave field synthesis and its regularization by analyzing the noise features that arise during wave field synthesis based on non-ideal point sources and proposing a novel structure for regularized point source construction. It is shown that the quality of plasmonic wave field synthesis based on the Huygens' principle is greatly improved with the proposed design of a structure that generates a unit point source. Practical field synthesis examples involving plasmonic focusing and Airy beams are presented in support of the © 2017 Optical Society of America proposed design.

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Optical wave field synthesis is a fundamental issue in optics, with various methods having been proposed to achieve it. It is accepted that, in theory, a discrete array of optical point sources can generate arbitrary wavefronts, something which is known as the Huygens' principle [1]. It posits that an optical wave can be considered to be a point source array that generates secondary waves. For the generation of arbitrary wavefronts, the point sources are complex-modulated, and the distance between adjacent point sources should be smaller than the constraint given by the Nyquist sampling theory, which is typically given by a half-wavelength.

Previous research on optical field synthesis has focused primarily on amplitude, phase, or complex modulation; research on the point sources themselves is relatively rare. In terms of wavefront modulation, spatial light modulators (SLMs) are a representative example of field synthesis engineering. Though the ultimate goal is to produce a unit pixel that is a half-wavelength in size, in practice, SLM pixels operate on a scale of multiple wavelengths. A strong approximation is taken in most wave field synthesis mechanisms due to the physical limitation of the unit pixel size reduction. Recently, metasurfaces with subwavelength pixel sizes are actively studied for optical field synthesis, including metasurface-based three-dimensional (3D) field synthesis [2-5] and plasmonic two-dimensional (2D) field synthesis [6-11]. Most of these previous studies have been devoted to wavefront modulation, but research on point sources is rare, even though the elementary radiation pattern of point sources can have a nonnegligible effect on field synthesis quality. Here, we would like to argue that there is strong approximation on the radiating profile of each point source in most field synthesis schemes, and these inevitably generate optical background noise, even though the modulation of each point source is supposed to be perfectly complex-valued.

In this Letter, we investigate Huygens' plasmonic wave field synthesis and the noise features associated with point source radiation patterns. A novel plasmonic structure for the generation of near-ideal point source array is proposed. The perfect synthesis of an optical wave field can be realized based on the Huygens' principle. In a 3D situation, for fixed frequency f, the optical wave field generated from an array of point sources with complex amplitudes is represented as

$$\vec{\psi}_{\text{synt}}(\vec{r},\omega) = \sum_{p,q} a_{p,q} \vec{G}_{p,q}(\vec{r},\vec{r}_{p,q}).$$
(1)

As in the holographic pickup process, each complex amplitude $a_{p,q}$ picks a local field profile at the position as $a_{p,q} = \psi_{\text{target}}(x = p\Lambda, y = q\Lambda)$, where Λ is the sampling interval and the target field $\psi_{\text{target}}(x, y)$ is supposed to be bandlimited.

If each radiating profile can be set to a mathematically equal point source function $\vec{G}_{n,m} = \vec{G}_{reg}$, then the generated field $\vec{\psi}_{synt}$ can be represented by the 2D convolution of the sampled pulse train of the original field and the unit dipole field, given as

$$\vec{\psi}_{\text{synt}} = \left[\sum_{n,m} \psi_{\text{target}}(\vec{r}) \delta(\vec{r} - \vec{r}_{n,m})\right] *_{x,y} \vec{G}_{\text{reg}}(\vec{r}, 0).$$
(2)

In order to reconstruct the perfect target field such that $\vec{\psi}_{synt} = \eta \vec{\psi}_{target}$, we need to state the mathematical condition of the unit point source for perfect reconstruction in the angular spectrum (Fourier) domain. As seen in Figs. 1(a) and 1(b), the angular spectrum of the ideal point source radiation pattern with polarization α , $\vec{G}_{ideal,\alpha}$, is explicitly given as

$$\mathcal{F}_{x,y}\{\vec{G}_{\mathrm{ideal},\alpha}\}\big|_{z=0} = \vec{\tilde{G}}_{\mathrm{ideal},\alpha} = \eta U_{k_0}(\vec{k}_{\perp})\hat{\alpha}, \qquad (3)$$

where $U_{k_0}(k_{\perp})$ is an indicator function of the transverse wavevector having the value 1 for $|k| \le k_0$ and the value 0 for otherwise, and $\hat{\alpha}$ stands for a fixed polarization unit vector parallel to the *xy*-plane.

In general, we cannot obtain the ideal point source array set $\vec{G}_{p,q} = \vec{G}_{ideal,\alpha}$. However, we need to secure the homogeneity of radiation over the point sources. Here, this homogenization is referred to *regularization of point source array*, i.e., $\vec{G}_{n,m} = \vec{G}_{reg}$.

As illustrated in Figs. 1(c) and 1(d), in general, a noise field is caused by the distortion of radiating patterns. In the analysis, it is assumed that the sampling of complex amplitude is perfect, but the noise field is induced only by the non-ideal point sources. The focus of the analysis is the effect of the non-ideal point source in the field synthesis. The noise field is then defined as the residual field over the original target field given by

$$\vec{\psi}_{\text{res}} = \vec{\psi}_{\text{synt}} - \eta \vec{\psi}_{\text{target}} = \sum_{p,q} a_{p,q} \cdot [\vec{G}_{p,q} - \vec{G}_{\text{ideal},\alpha}](\vec{r}, \vec{r}_{p,q}).$$
(4)

However, it is worth noting that the gratings that form a periodic array of non-ideal, but identical, point sources can generate nearly collimated and noise-reduced wave field, even



Fig. 1. (a, b) Schematics of a radiating profile of an ideal point source and Huygens' optical field synthesis with the ideal point sources (red). (c, d) Schematics of a radiating profile of a non-ideal point source and Huygens' optical field synthesis with non-ideal point sources (blue). Each arrow represents a point source. The residual field $\vec{\psi}_{\rm res}$ creates background noise.

though their single radiation unit generates non-ideal point sources.

2D plasmonic field synthesis is much simpler than 3D cases because only the TM polarization 2D scalar field describes all of the information of plasmonic field distribution. Moreover, assuming that the plasmonic radiation field from the unit magnetic dipole moment $\vec{m}_{\theta_0} = \delta(\vec{\rho})(\cos \theta_0 \hat{x} + \sin \theta_0 \hat{y})$ is given as $G_{\vec{m}_{\theta_0}} = (k_0/4i)H_0^{(2)}(k_0\rho)\cos(\theta - \theta_0)$, where $H_0^{(2)}(x)$ is a Hankel function of the second kind, we can get the Fourier transform of the radiation profile to the near-ideal unit rectangular function $\mathcal{F}_x\{G_{\vec{m}_0}\}|_{y=0} = \tilde{G}_{\vec{m}_0} = \eta U_{k_0}(k_x)$, where $U_{k_0}(k_x)$ is an indicator function of transverse wavenumber having the value 1 for $|k_x| \leq k_0$ and the value 0 for otherwise. In this case, sampling the target field using the *x*-polarized magnetic dipoles can perfectly generate any plasmonic field. For example, a slit parallel to the *x*-axis can be served as the magnetic dipole source of plasmonic field, that is, $G_{ideal,2D} = G_{\vec{m}_0}$.

Researchers have suggested many methods for plasmonic field synthesis to date [6–11], and we can choose a double-lined slit array as a representative method [9–11] and illustrate its structure in Figs. 2(a) and 2(c). Its mechanism relies mainly on the Huygens' principle through equidistant sampling of the target field. This type of structure can generate an arbitrary plasmonic wavefront because the geometric phase induced by the rotation angles of slits (θ_1 , θ_2), together with the superposition of scatterers, provides complex amplitude modulation. However, as we discussed above, noise is non-negligible because the two slits in each unit cell are distant from each other, as seen in Fig. 2(c). The slits generate a plasmonic field independently because each slit is distant from the neighbor slit, and the scattering is not strong enough to produce high-order scattering



Fig. 2. (a, c) Schematics of a conventional double-lined slit array. (b, d) Schematics of the proposed structure, regularizing each unit cell of the shifted slit array. The fields radiated by the regularized unit cells collectively form a plasmonic wavefront. Both cases assume the incident light left circularly polarized plane wave along the +z-axis. Each slit scatters light into the form of a plasmonic field with a radiating profile of magnetic dipoles. Here, dimensions of the metal-clad waveguide are given as $c_x = 100 \text{ nm}$, $c_z = 390 \text{ nm}$, w = 100 nm, $L_{\text{prop}} = 2 \text{ um}$, and $t_{\text{top}} = t_{\text{bottom}} = 80 \text{ nm}$.

terms. For the perfect synthesis of a plasmonic field, we engage in the regularization of the single slit, of which rotation angle is θ_n , in each cell by introducing a metal-clad plasmonic waveguide, as depicted in Figs. 2(b) and 2(d). The essential elements in the fabrication are the sidewalls and supporting upper metal plates. Since the core of the waveguide can be replaced into a dielectric medium, a metal deposition after milling of the dielectric film on the lower metal film can form sidewalls and upper-metal film simultaneously.

The working principle of the proposed structure can be explained as follows. The metal cladding isolates each unit cell from its neighbors. Its size is small and anisotropic enough so that only one fundamental mode can propagate through the waveguide. The electric field of the propagating mode is mainly z-polarized, and the magnetic field is mainly x-polarized. Therefore, for an incidence of left-circular polarized light as in Fig. 2(b), the relative coupling coefficient is represented by $c_n = \cos \theta_n \exp(-i\theta_n)$. Note that the circular polarization of the incident wave additionally induces a geometric phase, $\exp(-i\theta_n)$. The position of each slit is y-shifted as much as d_n ; then an additional phase exp $(-i2\pi d_n/\lambda_{\text{eff}})$ is introduced, where $\lambda_{\rm eff}$ is effective wavelength of the fundamental mode of the metal-clad waveguide. In the final stage, the radiation from the waveguide to the half-infinite metal plane is also approximated using the magnetic dipole radiation, $G_{\text{reg}} \approx G_{\vec{m}_0}$, because the waveguide supports the only propagating mode with localized and mainly the x-polarized magnetic field. The verification of this is presented in Fig. 3. The complex amplitude of the radiation for a unit cell is $a_n = \xi c_n =$ $\xi \exp(-i\theta_n - i2\pi d_n/\lambda_{\text{eff}}) \cos \theta_n$, where ξ is a proportional constant. The overall radiation pattern for a unit cell of the proposed structure follows $a_n G_{\text{reg}} \approx a_n G_{\vec{m}_0} \approx \xi c_n G_{\text{ideal,2D}}$; thus, complex modulation and regularization are simultaneously accomplished by the proposed structure.

In contrast, the radiation pattern of a unit cell of the conventional double-lined slit array is $E_{z,dls} \approx [G_{\vec{m}_{\theta_1}} \exp(-i\theta_1) + G_{\vec{m}_{\theta_n}} \exp(-i\pi \cos \theta - i\theta_2)]$. The double-lined slit array works



Fig. 3. (a, c) Radiation field distributions of a unit cell of doublelined slit array for $\theta_1 = -30^\circ$, $\theta_2 = 60^\circ$. (b, d) Radiation field distributions in the proposed structure for $\theta = 0^\circ$, d = 0 nm. The purple dashed lines indicate y = 0 um. (e) Waveguide mode efficiently produces a plasmonic field.

as desired under paraxial launching, but the radiation profile of a unit cell is irregular and varies in parameter space (θ_1, θ_2) .

We now present numerical results as a proof-of-concept. 3D full-field simulations were carried out using CST studio SUITE. Analytical modeling of a plasmonic field generated by double-lined slit arrays and ideal point source arrays was performed using MATLAB [9–11]. The wavelengths were fixed at 980 nm, and the metal was assumed to be gold. All dimensions are illustrated in Figs. 2(c) and 2(d). The sampling period satisfies the Nyquist sampling rate, $\Lambda \leq \lambda/2$. The structural parameters are chosen for obtaining cell isolation and a single propagation mode ($c_x < L/2, c_y \ll \lambda$). Note that if multimode excitation were involved, the modulation could not be expressed in any simple form. In Fig. 3, we analyze the effect of the regularization process. For the case of the double slits parameterized by $(\theta_1, \theta_2) = (-30^\circ, 60^\circ)$, the analytical radiation pattern of the two slits is presented in Fig. 3(a). In Fig. 3(c), the 3D full-field simulation result confirms the analytical model. On the other hand, the proposed structure successfully regularizes the radiation field distribution, as we can see from Figs. 3(b) and 3(d). Figure 3(e) shows that the metalclad waveguide launches a plasmonic field to the half-infinite metal plane. Figures 4 and 5 show the examples of Huygens' plasmonic field synthesis that prove the validity of the proposed structure in comparison with a conventional non-regularized double-lined slit array. Note that the sampling rate of each case is fixed at the inverse of half of the wavelength, and the number of sample points is 61.

Figure 4 depicts a focused plasmonic field to demonstrate the effect of regularization on the generation of a plasmonic field. The maximum intensity at the focal point can be generated by back-propagation:

$$a_n = \overline{G_{\vec{m}_0}(\theta)} = \overline{H_0^{(2)}\left(k_{\text{SPP}}\sqrt{f^2 + x^2}\right)} \cdot \left(f/\sqrt{f^2 + x^2}\right),$$
(5)



Fig. 4. Numerical results of SPP focusing at y = 7um with (a) the proposed structure and (b) the double-lined slit array. (c) Analytical dipole modeling produces a plasmonic field with a sinc function profile. (d) Amplitudes of the simulated fields at the focal line are compared. Each blue dashed line on (a), (b), and (c) depicts the corresponding focal line.



Fig. 5. Plasmonic Airy beams constructed by (a) the proposed structure and (b) the double-lined slit array. (c) Analytical dipole modeling produces a plasmonic Airy beam. (d) Amplitudes of the simulated fields at y = 7 um are compared. Each blue dashed line on (a), (b), and (c) depicts y = 7 um line.

where f is the distance between the reconstruction line and the focal line, and k_{SPP} refers to the complex wavenumber of surface plasmon polaritons. The overbar notation indicates complex conjugate operation. On the other hand, the sampling profile also implies that the generated field should be sinc(NA $\cdot k_0 y/\pi$), which is, as expected, the Fourier transform of the rectangular function. Analytical dipole modeling perfectly illustrates this situation in Fig. 4(c). We can use the profile to achieve diffraction-limited focusing with minimal noise, as in Fig. 4(a). The estimated signal-to-noise ratio (SNR) is about 54. The conventional double-lined slit array focuses near the desired position, but it is slightly shifted, and background noise also appears with a low SNR of approximately 0.14 as in Fig. 4(b). Regularized dipole radiation is realized by the proposed structure, and 3D full-field simulation confirms that tight and bell-shaped focusing is produced via the regularization process, as seen in Fig. 4(d). It is shown that very accurate plasmonic field distribution is achieved in this case. Figure 5 displays another example of Airy plasmonic field generation via meta-slit and its regularized version:

$$a_n = \text{Airy}((x - x_0)/a).$$
 (6)

The parameters are fixed at $x_0 = 10$ um, $a = 1.1\lambda$. This distribution of dipole amplitude should generate a nondiffracting plasmonic Airy beam as in Fig. 5(c). As seen in Fig. 5(b), the conventional method distorts the main lobe of the synthesized field distribution severely by background noise. In contrast, the proposed structure generates a noise-reduced non-diffracting profile for the Airy plasmonic beam as in Fig. 5(a). The finite length of the double-lined slit array blurs the outer side-lobes. The amplitudes of the simulated fields at the same reconstruction plane, y = 7 um, are compared in Fig. 5(d). It is shown that the distortion near the main lobe is greatly reduced by the regularization process.

In conclusion, we have investigated the effect of the regularization of point sources in the generation of arbitrary wavefronts with extremely low background noise and hope that it can be applied to the development of advanced meta-surfaces and SLMs.

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