Optimal nonmonotonic convergence of the iterative Fourier-transform algorithm

Hwi Kim and Byoungho Lee

School of Electrical Engineering, Seoul National University, Kwanak-Gu Shinlim-Dong, Seoul 151-744, South Korea

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The increase of the monotonic convergence rate is an important issue for iterative Fourier-transform algorithms. However, the steepest monotonic convergence of the iterative Fourier-transform algorithm does not always promise an optimal solution in the design of a diffractive optical element. The optimal nonmonotonic convergence of the iterative Fourier-transform algorithm is investigated by employing a microgenetic algorithm. The proposed hybrid scheme of the iterative Fourier-transform algorithm and the microgenetic algorithm show nonmonotonic convergence, and this results in a superior design.

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The iterative Fourier-transform algorithm (IFTA) is the most popular algorithm for the design of a diffractive optical element (DOE).[1] The DOE design problem does not allow analytic approaches, and an exact solution may not exist.[1] The degree of freedom is important in a nonlinear problem as it is in DOE design. Commonly, amplitude and phase freedoms are employed in DOE design.[2,3] In this Letter the relaxation parameter[1,4] is recognized as an effective degree of freedom. With the same condition, IFTAs with different relaxation parameters give considerably different results. In this Letter we propose a method of finding the optimal phase profile by clever use of the relaxation parameter.

In general, monotonic convergence of the IFTA is adopted as a matter of course. However, the steepest monotonic convergence of the IFTA[4] does not always promise the optimum solution for nonlinear problems such as DOE design. Through a monotonic convergence, the IFTA may reach a local optimum near the starting point. As is well known, the relaxation parameter is a key factor in controlling the convergence rate of the IFTA.[1,4] Generally, the relaxation parameter is considered constant or a single number determined at each stage to induce the steepest descent convergence.[4] However, we take a new angle on the relaxation parameter. In this Letter the set of relaxation parameters $\{\lambda_n | n = 1, 2, \cdots, N\}$, the so-called relaxation parameter vector with dimensionality N equal to the previously given number of iterations of the IFTA, will be optimized by the microgenetic algorithm (μGA).[5] It is expected that the IFTA with an optimized relaxation parameter vector will show nonmonotonic convergence.

Figure 1 shows the flow chart of the proposed hybrid scheme. As shown in Fig. 1, the IFTA loop (denoted in the figure by IFTA) is embedded in the μGA as a subroutine to generate the fitness value. A chromosomal individual of the μGA is relaxation parameter vector $\{\lambda_n | n = 1, 2, \cdots, N\}$ (denoted by x). In our μGA, float point coding is adopted.[6] The μGA is a small-population-size genetic algorithm (five individuals are usually maintained). The main strategy of the μGA is to restart consecutively each time local convergence is achieved through an internal crossover loop (indicated in the left-hand portion of Fig. 1), i.e., $|f_{\min} - f_{\mathrm{avr}}| < \delta$, by saving the elite individual and newly creating the whole population by adaptive mutation (indicated in the right-hand portion of Fig. 1). To evaluate the fitness value of each relaxation parameter vector the IFTA subroutine starts from scratch and is iterated just N times with each relaxation parameter vector applied. As shown in Fig. 1, the internal genetic operation consists of crossover, mutation, and elitist approaches for selection. For the crossover a simple linear combination of two individuals is used in which the linear combination coefficient and the crossover probability are tuned to 0.4 and 0.5, respectively (see Chap. 6 of Ref. 6). For the mutation an adaptive mutation operator for float point coding is implemented in which the mutation probability and the system parameter determining the degree of nonuniformity are tuned to 0.05 and 5, respectively (see Chaps. 5 and 6 of Ref. 6). The μGA was previously used in the design of DOEs reported in Ref. 5, in which the phase profile of the DOE was directly optimized by the μGA. In our approach, however, the phase profile

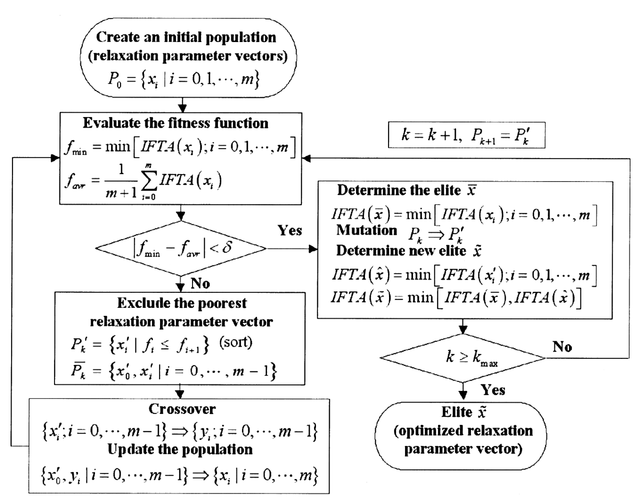

Fig. 1. Flow chart of the proposed hybrid scheme of the IFTA and the μGA.

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of the DOE is obtained through an embedded IFTA subroutine. It should be understood that the IFTA itself is optimized by the μGA.

In this Letter a new version of IFTA recently developed by Kim *et al.*[2] is presented that takes the form

$$\overline{F}_n = \begin{cases} \lambda_n F_0 \exp(j\psi_n) + \left\{1 - \lambda_n - \frac{2\lambda_n}{\pi} \tan^{-1}\left[\frac{|F_n(x,y)| - F_0(x,y)}{F_0(x,y)}\right]\right\} F_n + \lambda_n \alpha_D \nabla^2 |F_n| \exp(i\psi_n) & \text{for } (x,y) \in S \\ F_n & \text{for } (x,y) \notin S \end{cases}, \tag{1a}$$

$$F_{n+1} = \mathrm{Fr} D_{\mathrm{DOE}} \mathrm{Fr}^{-1}(\overline{F}_n), \tag{1b}$$

where F_0, F_n, ψ_n, λ_n, and α_D denote the target image amplitude, the diffraction field at the nth iteration stage, its phase profile, the relaxation parameter at the nth iteration stage, and the regularization parameter, respectively. Fr, Fr^{-1}, D_{DOE}, and S represent the Fresnel transform, its inverse transform, the amplitude operator in the DOE plane, and the signal area, respectively. For general cases including gray images, the definition of the uniformity is defined as

$$\text{uniformity} = \frac{||F| - \sqrt{I_0} + \epsilon|_{\max} - ||F| - \sqrt{I_0} + \epsilon|_{\min}}{||F| - \sqrt{I_0} + \epsilon|_{\max} + ||F| - \sqrt{I_0} + \epsilon|_{\min}}. \tag{2}$$

Bias parameter ϵ should be selected carefully. For the definition of the uniformity [Eq. (2)] to correctly indicate the uniformity of the solution, ϵ must be selected to make the inner terms positive ($\epsilon > \sqrt{I_0} - |F|$) in the whole signal region. In the simulation ϵ is set to 0.4. The fitness function of the μGA to be minimized is designed as

$$E(F) = \frac{\iint_S (|F| - \sqrt{I_0})^2 \mathrm{d}x\mathrm{d}y}{\iint_S I_0 \mathrm{d}x\mathrm{d}y} + w \frac{||F| - \sqrt{I_0} + \epsilon|_{\max} - ||F| - \sqrt{I_0} + \epsilon|_{\min}}{||F| - \sqrt{I_0} + \epsilon|_{\max} + ||F| - \sqrt{I_0} + \epsilon|_{\min}}, \tag{3}$$

where the first term is the mean-square error and the second term is the uniformity. Fitness function $E(F)$ consists of a combination of two terms through weight parameter w. In the simulation w is set to 4. Note that the evaluation of the uniformity is done only in the signal area.

For the simulation the target intensity distribution is selected as shown in Fig. 2. We restrict the phase profile of the DOE to be symmetric to reflect the symmetry of the target image. The regularization parameter α_D in Eq. (1) is determined as 0.2.[2,3] Length N of the relaxation parameter vector is selected to be 100. The range of all the relaxation parameters is set to $[-200, 200]$. All the relaxation parameters are set to 1 for the conventional simple IFTA scheme to be compared with the proposed scheme. We purposely include the default relaxation parameter vector $\{\lambda_n | \lambda_n = 1, n = 1, 2, \cdots, N\}$ in the initial population of the μGA. This initial setting and the elitist selection rule of the μGA guarantee a superiority of the proposed scheme.

Comparisons of the results of the proposed hybrid scheme and the simple IFTA are shown in Figs. 3 and 4. Since the simple IFTA reaches the stagnation point completely, after 1500 iterations, in the case of the simple IFTA, the result at the 1500th iteration stage is selected. Figures 3(a) and 3(b) show the intensity distributions generated by the DOEs obtained by the proposed hybrid scheme (at the 100th iteration stage) and the simple IFTA (at the 1500th iteration stage), respectively. Comparing the results, we find that the proposed scheme (which results in a diffraction efficiency of 85.7% and a uniformity of 0.113) is superior to the simple IFTA (which results in a diffraction efficiency of 82.4% and a uniformity of 0.401) in both diffraction efficiency and uniformity. Figures 4(a) and 4(b) show the convergence curves in uniformity and diffraction efficiency of the IFTA in the proposed scheme and the simple IFTA, respectively. Since the simple IFTA reaches the stagnation point completely by 1500 iterations, to clearly compare the performance of the proposed scheme and the simple IFTA we purposely extend the total iteration number of the IFTA in the proposed scheme to 1500 times with all the relaxation parameters equal to 1 after the previous 100 iterations with the optimized relaxation parameter vector. As can be seen from Fig. 4 both uniformity and diffraction efficiency improve greatly with the proposed scheme (as indicated by the solid

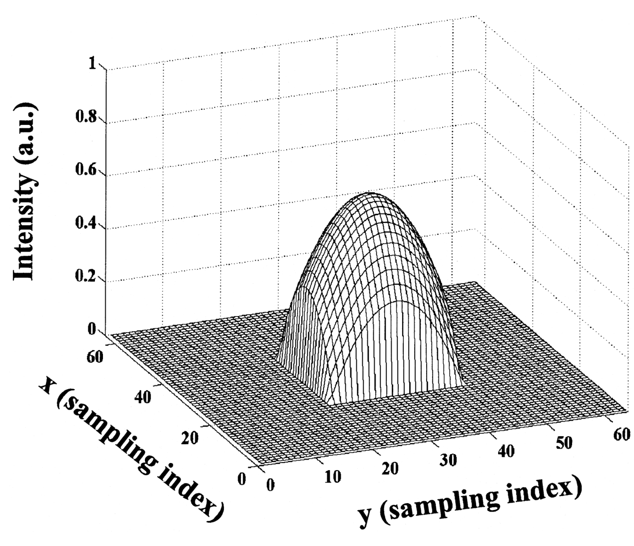

Fig. 2. Target image (intensity distribution) of an example of a DOE design.

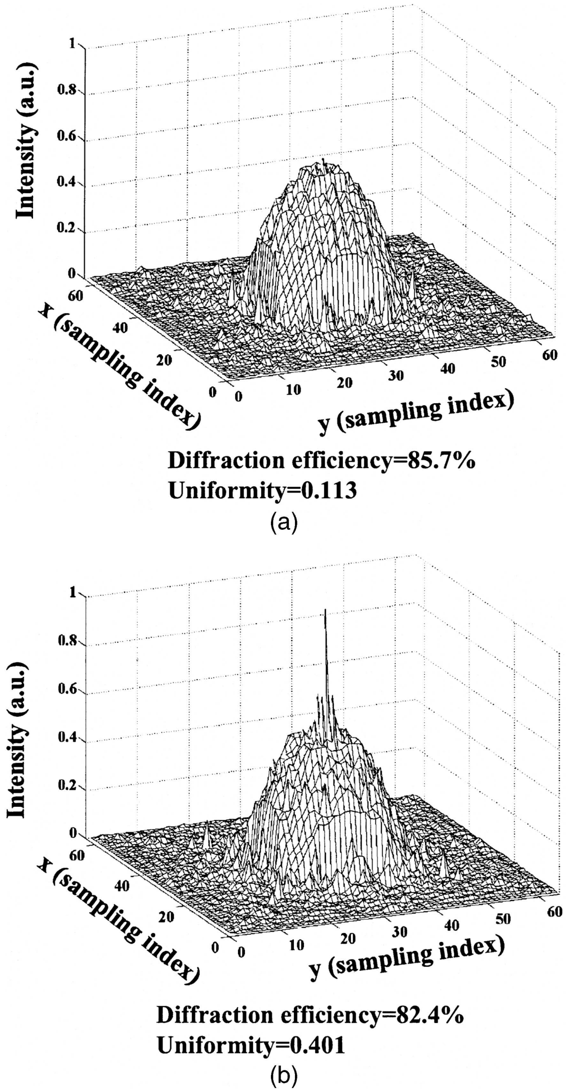

Fig. 3. (a) Intensity distribution of the diffraction image generated by the DOE designed by (a) the proposed hybrid scheme and (b) the simple IFTA.

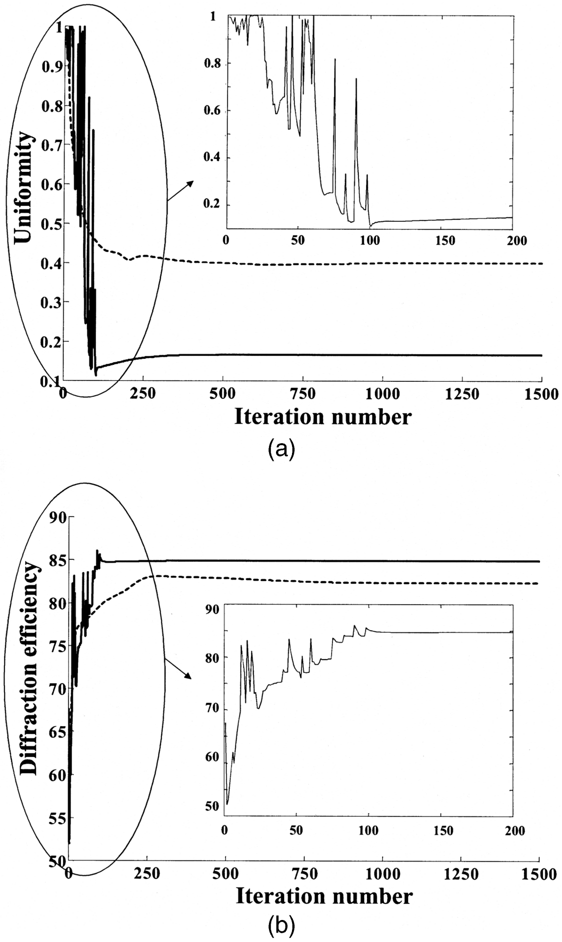

Fig. 4. Comparison of the convergence feature of (a) uniformity and (b) diffraction efficiency between the simple IFTA (dotted curves) and the proposed hybrid scheme (solid curves).

curves) through nonmonotonic convergence. In addition, the μGA tries to make the previously given 100th stage the optimum point. Thus the best-quality solution is obtained at the 100th iteration stage (i.e., the optimum point) as shown in the enlargements at the left. With only tens of iterations of the μGA, the proposed scheme can produce a solution superior to that of a simple IFTA since the μGA with the elitist selection rule converges quickly in a few early iterations. The small size of a chromosomal individual lessens the computation load of the internal genetic algorithm operators. Then the proposed hybrid scheme has advantages of lower computation cost than the conventional usage of a genetic algorithm or simulated annealing and superiority of the resulting solution compared with the simple IFTA. Practically, we can drastically reduce the computation time by using the parallel implementation of the μGA.[7]

In conclusion, we have shown that the optimal nonmonotonic convergence of the proposed hybrid scheme guarantees a superior solution to the monotonic convergence of the conventional IFTA.

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