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Diffractive Optical Element with Apodized Aperture for Shaping Vortex-Free Diffraction Image

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In the diffraction image formed by a diffractive optical element with a coherent laser beam, optical vortices appear. Preventing optical vortices in a diffraction image is an important issue particulary in beam shaping application. In this letter, a novel design of diffractive optical elements for generating vortex-free diffraction images is proposed. It is manifested that the key point of preventing optical vortices is the apodization of the aperture of a diffractive optical element. The optimization of the aperture apodization process based on the genetic algorithm is discussed. [DOI: 10.1143/JJAP.43.L1530]

KEYWORDS: diffractive optical element, optical vortex, aperture apodization

A diffractive optical element (DOE) is a device that forms a diffraction image on a specified image plane. DOEs are used in material processing, optical information processing, optical interconnections and displays.[1,2] It is well known that when a coherent laser beam such as a He-Ne laser beam is used as a light source, optical vortices (dark holes of zero intensity) appear in the diffraction image generated by DOEs.[3] However, many beam shaping applications require the generation of diffraction images with a reduced number of optical vortices or without optical vortices. Optical vortices can be simply removed from a diffraction image by statistically averaging out temporal diffraction images using a partially coherent light source. However, any statistical averaging-out technique is not considered in this study. The coherence of the optical field is assumed to be complete. The problem of preventing optical vortices in coherent diffractive beam shaping was discussed in previous reports. Aagedal *et al.*[3] introduced a few important concepts such as the initial image phase distribution, the hard clip operation in the iterative Fourier transform algorithm (IFTA) and the phase dislocation. This letter shares the same framework as that of Aagedal *et al.* They also proposed the soft coding method to prevent optical vortices in the obtained diffraction image. However, they observed that the soft coding method could not completely eliminate optical vortices. In this letter, a novel method for preventing optical vortices in diffractive beam shaping is proposed.

The design of a DOE involves the determination of the phase profile of the DOE to form a target diffraction image on an output plane. The IFTA is the most popular algorithm for this purpose.[1–5] Practically, DOE calculation is performed on a discrete computation grid with the aid of the fast Fourier transform (FFT). Thus, we can control only a set of spatial sampling points of the diffraction field on the output plane. In the interval between any two adjacent spatial points, the field intensity may show random fluctuation.[1,3] This condition allows optical vortices to be generated in the diffraction image during the IFTA process. It is well known that various singular phase structures appear around optical vortices.[3] Let the complex amplitude function of a diffraction field generated by a DOE be $F(x, y)$. At the point of zero intensity, i.e., $|F(x, y)| = 0$, a definite phase value, $\arg(F(x, y))$, does not exist. This phase singularity is termed as phase dislocation. Phase dislocation indicates a singular point of the unwrapped phase distribution of the diffraction field. Figure 1 shows an example of a conventional DOE design. The IFTA scheme used is thoroughly described in refs. 4 and 5. A random phase distribution was adopted as the initial phase profile of the DOE in this example. As seen in Fig. 1(a), the diffraction image includes many optical vortices that look like small dark holes. The phase profile of the designed DOE with a conventional circular aperture is presented in Fig. 1(b).

On the other hand, it is expected that if there is no phase dislocation in the specified signal area of the output plane,

(a)

(b)

Fig. 1. (a) Diffraction image and (b) phase profile of conventional DOE without aperture apodization.

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optical vortices will not appear in the signal area. In other words, if the diffraction field has regularity in the signal area, we can expect a vortex-free diffraction image on the output plane in principle. According to ref. 3, the cause of generating phase dislocations during the conventional IFTA process is the internal hard clip operation of the IFTA. The hard clip operation indicates the process of abruptly replacing the amplitude of the optical field with a fixed amplitude profile to satisfy the boundary condition on the input plane at every iteration stage. Although the non-linearity of the IFTA makes it difficult to perform a precise analysis on the relationship between the hard clip operation and the generation of phase dislocations, it is shown in ref. 3 that the hard clip operation destroys the regularity of the phase distribution of the diffraction field, which is referred to as the image phase distribution, and induces phase dislocations during the IFTA process. Hence, a possible strategy for obtaining a vortex-free diffraction image is to conserve the regularity of the image phase distribution during the IFTA process. To achieve this, an effective method for minimizing the hard clip effect at every iteration stage is required. In this letter, it is shown that the aperture apodization of the DOE is effective for minimizing the hard clip effect and a novel design method for a DOE with an optimally apodized aperture for generating a vortex-free diffraction image is proposed. The devised design procedure is composed of two steps: the optimization of the aperture apodization and IFTA processes with the apodized aperture.

The first step is optimally apodizing the aperture of the DOE. Figure 2(a) shows the flow chart of the first step. Let the target image and initial image phase distribution be $F_0(x,y)$ and $\Omega_0(x,y)$, respectively. $F_0(x,y)$ is multiplied by the exponentiation of $j\Omega_0(x,y)$ to construct the complex target image $F_0(x,y)\exp(j\Omega_0(x,y))$. Continuously the complex target image is inversely transformed through the inverse Fresnel transform (denoted as $IFr(\cdot)$)[4] to generate the inverse image $A(u,v)$ on the input plane as

$$A(u,v) = IFr[F_0(x,y)\exp(j\Omega_0(x,y))]. \tag{1}$$

We can see that if an ideal optical field with the same complex amplitude as $A(u,v)$ exists on the input plane, the diffraction image obtained by the forward Fresnel transform[4] of $A(u,v)$ is definitely the same as $F_0(x,y)$. However, in the DOE design, no amplitude freedom is allowed on the input plane. The amplitude profile of the complex field on the input plane must be the same as that of the incident beam. At a portion of the initial inverse image with a small amplitude, a big difference between the amplitude of the incident beam and that of the inverse image exists. The hard clip effect indicates that the big difference induces phase dislocations in the diffraction field. However, we note that the aperture shape, i.e., the aperture apodization, is an important degree of freedom. In the proposed scheme presented in Fig. 2(a), the aperture apodization process is devised to exclude the portion of the initial inverse image with a small amplitude. We think that this work will result in the weakening of the hard clip effect at every iteration stage during the IFTA process. Practically, the aperture apodization function $\Gamma(u,v)$ is defined as

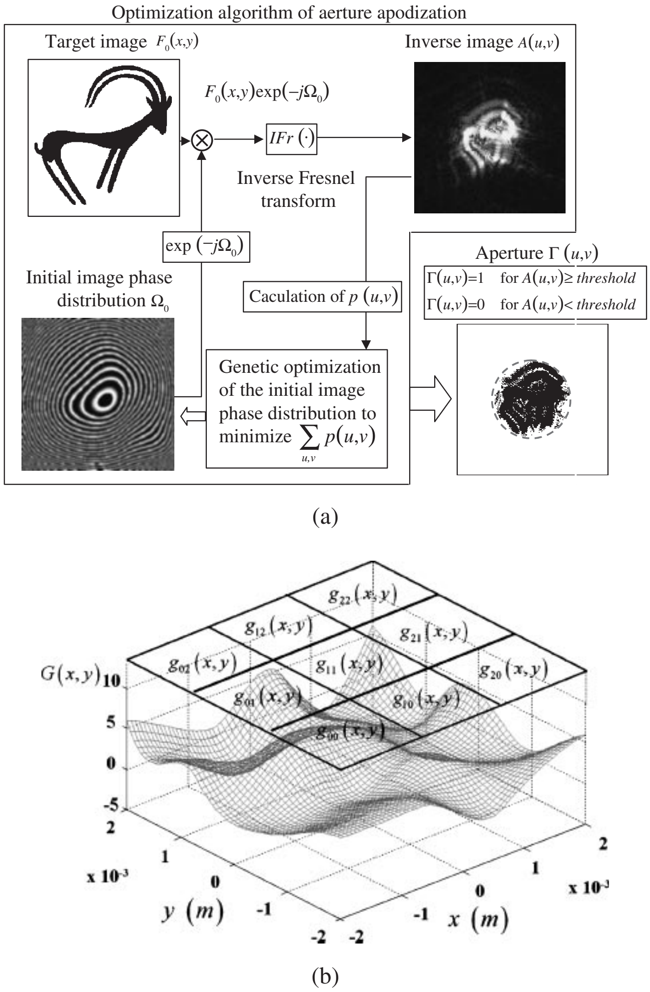

Fig. 2. (a) Flow chart of generating optimal aperture apodization and (b) curved surface of 3×3 polynomial composite.

$$\Gamma(u,v) = 1, \quad \text{for } |A(u,v)| \geq \text{threshold}, \ (u,v) \in M \\ = 0, \quad \text{otherwise} \tag{2}$$

where M denotes the cross section of the incident optical wave. In principle, the *threshold* indicates a minimum amplitude contributing to the construction of the aperture, which should be determined simultaneously to sufficiently weaken the hard clip effect or prevent the formation of optical vortices in the diffraction image. With the *threshold*, the level of the hard clip effect can be controlled. Moreover, optical vortices will be eliminated in the diffraction image if the aperture apodization process with an optimum *threshold* minimizes the hard clip effect sufficiently. Although the *threshold* is rather empirically determined depending on each target image, it is advisable to select a small *threshold* to achieve apodization in a wider aperture area. We empirically know that about 1/10 of the highest amplitude of the inverse image is the proper optimum *threshold*. It is expected that the apodization of the DOE aperture of eq. (2) will conserve the regularity of the initial image phase distribution and prevent the occurrence of phase dislocations in the diffraction image during the IFTA process. This is the reason why the aperture apodization of the DOE prevents the formation of optical vortices in the diffraction image. Practically in this letter, the *threshold* (1/10) was deter-

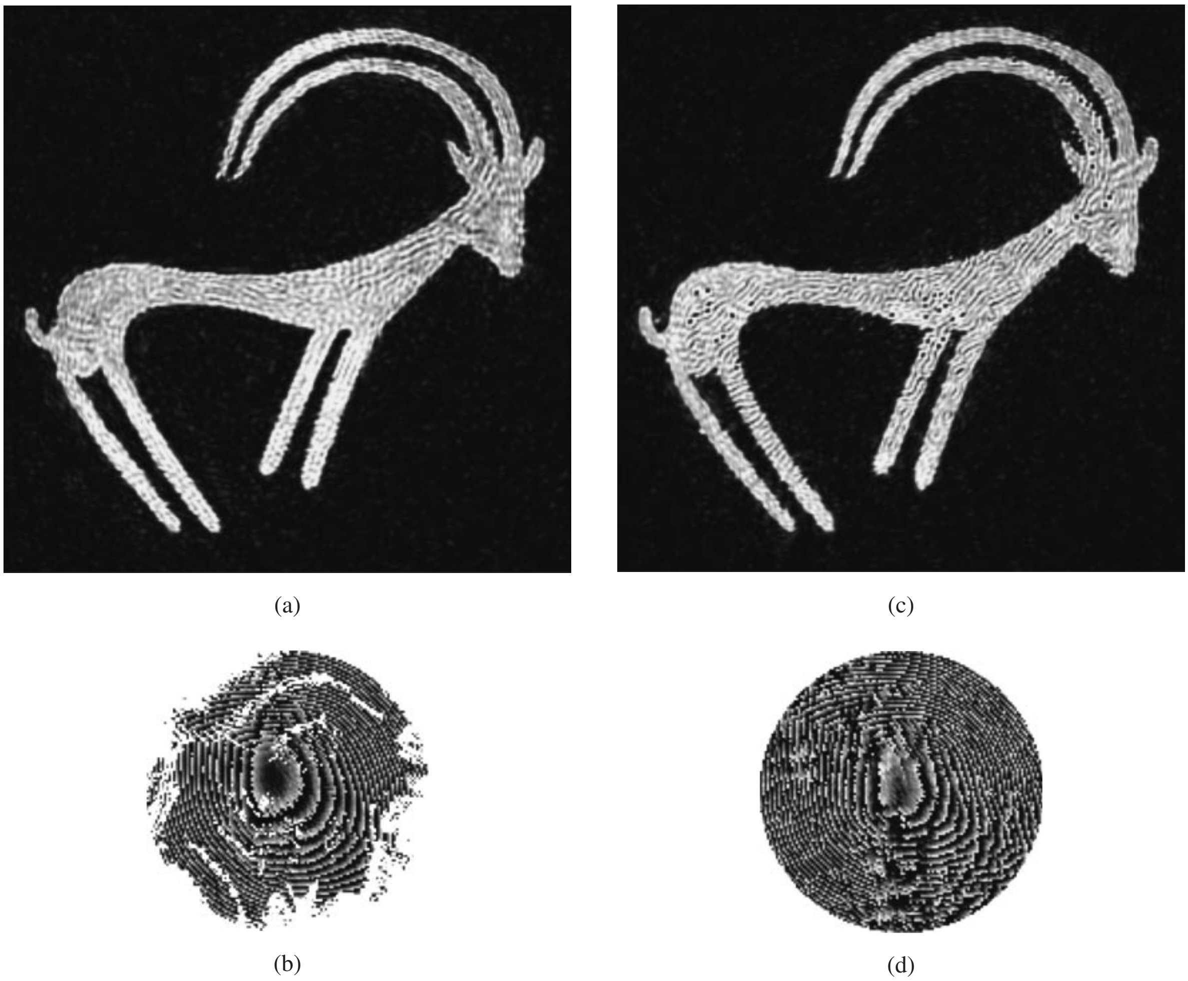

Fig. 3. (a) Diffraction image and (b) phase profile of DOE with apodized aperture, and (c) diffraction image and (d) phase profile of DOE designed by soft coding.

mined to generate a vortex-free diffraction image at the first IFTA iteration stage for the inverse image obtained with a standard quadratic phase distribution.

In addition, it is strongly required to enlarge the area of the aperture $\Gamma(u,v)$. Enlarging the area of the aperture involves increases in both the transmission efficiency of the incident optical wave and the number of available phase sample variables for the design. In particular, the number of variables influences the accuracy of the resulting diffraction image. If we can optimize the initial image phase distribution to make the aperture shape fill the cross section M of the incident optical wave completely, we will obtain the ideal solution. In general, this ideal solution cannot be obtained. Thus, the optimization of the aperture apodization process should be considered. In the proposed scheme, as indicated in Fig. 2(a), the aperture apodization process is optimized to maximally reflect the incident beam cross section through the initial image phase distribution.

For the optimization of the aperture apodization process, the fitness function T is defined as

$$T = \sum_{u,v} p(u,v), \tag{3}$$

where the aperture indicator function $p(u,v)$ indicates the binary quantization of the amplitude of the inverse image $A(u,v)$ by a specific threshold value as follows.

$$p(u,v) = \begin{cases} 0, & \text{for } |A(u,v)| \geq \textit{threshold},\ (u,v) \in M \\ 1, & \text{for } |A(u,v)| < \textit{threshold},\ (u,v) \in M \\ 1, & \text{for } |A(u,v)| \geq \textit{threshold},\ (u,v) \notin M \\ 0, & \text{for } |A(u,v)| < \textit{threshold},\ (u,v) \notin M \end{cases} \tag{4}$$

It is noted that the aperture indicator function $p(u,v)$ in eq. (3) indicates both the portion of the inverse image with an amplitude less than the *threshold* inside M and that of the inverse image with an amplitude larger than the *threshold* outside M. Hence, the minimization of the objective function T will enlarge the portion of the inverse image with an amplitude larger than the *threshold* inside M, i.e., the aperture shape of the DOE, and reduce that of the inverse image with an amplitude larger than the *threshold* outside M.

Although there are many optimization algorithms that can be applied to optimize the aperture apodization process, in this letter, the conventional genetic algorithm (GA) with float point coding (FP coding)[6] is selected as the optimization tool. The chromosomal individual of the GA is the curved surface function representing the initial image phase distribution. An appropriate curved surface model should be selected to properly reduce the fitness function T to a minimum. In the proposed scheme, the following polynomial function Ω_0 is considered as the initial image phase distribution.

$$\Omega_0 = c(x^2 + y^2) - G(x, y) \tag{5}$$

Here, c is the parameter to be optimized and $G(x, y)$ is the smooth curved surface function. Equation (5) can be viewed as the sum of the standard quadratic phase distribution used in the soft coding method[3] and a perturbation term. As shown in Fig. 2(b), the perturbation term $G(x, y)$ is modeled by a smooth polynomial composite consisting of an array of elementary polynomial surface functions $z = g_{mn}(x, y)$ as

$$\begin{aligned} G(x, y) &= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} g_{mn}(x, y) \\ &= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left[\sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}^{mn}\left(\frac{x - x_m}{x_{m+1} - x_m}\right)^i\left(\frac{y - y_n}{y_{n+1} - y_n}\right)^j\right] \end{aligned} \tag{6}$$

where $(x, y) \in [x_m, x_{m+1}] \times [y_n, y_{n+1}]$ is the subdivision interval of the entire surface domain, and a_{ij}^{mn} are the parameters to be optimized. This subdivision concept is useful for the design of slowly varying smooth phase distribution.

The second step is the IFTA process for determing the phase profile of the DOE with the obtained apodized aperture. The corresponding curved surface function Ω_0 of eq. (5) is adopted as the initial image phase distribution in the IFTA process.

For the simulation, we used a 3×3 composite polynomial ($m = 0, 1, 2, n = 0, 1, 2$) as the perturbation term, as shown in Fig. 2(b). The *threshold* was set to be $1/10$ of the highest amplitude of the inverse image. In the left portion of Fig. 2(a), the image phase distribution optimized by the GA according to the described procedure is presented. In the right portion of Fig. 2(a), the inverse image $A(u, v)$ of the complex target image is shown. The aperture apodization $\Gamma(u, v)$ that is realized by the quantization of eq. (2) is presented in the right portion of Fig. 2(a). Figures 3(a) and 3(b) show respectively the diffraction image and the phase profile generated by the DOE with the apodized aperture. Comparing the diffraction image obtained using the proposed scheme with that obtained using the conventional design in Fig. 1 qualitatively, we can see that optical vortices are eliminated completely in the case of the proposed design. Figures 3(c) and 3(d) represent respectively the diffraction image and the phase profile generated by the DOE with a conventional circular aperture designed by the IFTA with soft coding.[3] The simple quadratic phase distribution $c(x^2 + y^2)$ was used as the initial image phase distribution in the soft coding IFTA.[3] As seen in Fig. 3(c), some optical vortices that look like dark holes appear in the diffraction image in the case of the soft coding method. As stated in ref. 3, the hard clip effect at a few latter iteration stages of the IFTA even with soft coding induces phase dislocations in the diffraction field inevitably.

Therefore, it was shown that the aperture apodization technique is considerably effective in eliminating optical vortices in the diffraction image. In conclusion, a novel IFTA-based DOE design scheme with an optimally apodized aperture was proposed for generating vortex-free diffraction images.

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